The Economic Value of Timing Higher Order (Co-)Moments in Bull and Bear Markets

The Economic Value of Timing Higher Order (Co-)Moments in Bull and Bear Markets*

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Abstract

We examine the ex-post performance of optimal portfolios with predictable returns, when the investor horizon ranges from one month to ten years. Due to the investor's ability to forecast shifts between bull and bear markets, predictability involves the risk premium, volatility and correlations, and may extend to third and fourth moments. We analyze three different equity portfolios data sets, each covering more than eight indexes, including commonly used US Industry and International Book-to-Market portfolios. Allowing for regimes improves portfolio performance for at least a subset of investment horizons and in all data sets. Despite substantial non-normalities in both the Industry and the book-to-market data sets, gains from predicting higher order moments obtain only in the latter. However, tracking and forecasting bull and bear markets turns out to improve realized portfolio performance more generally. The equally weighted strategy leads to lower ex-post performance measures than optimizing ones.

Key words: Equity market regimes, Bull and bear states, Return predictability, Skewness and kurtosis, Equity portfolio diversification.

JEL code: G11, F37, C22, C51.

1. **Introduction**

Risk-adjusted profits in active portfolio management derive from the ability of money managers to forecast returns out-of-sample. Among others, Bossaerts and Hillion (1999), Ang and Bekaert (2007) and Welch and Goyal (2008) have cast doubts on prevailing linear methods for asset returns predicting out-of-sample, which are reinforced by the inability of optimizing strategies to obtain out-of-sample gains relative to a naive, equally-weighted strategy (DeMiguel, Garlappi and Uppal, 2009).[1] Importantly, all these papers restrict

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[1] Allowing for a small amount of aversion to ambiguity about the distribution of stock returns also leads to an *out-of-sample* increase in Sharpe ratio, see e.g., Garlappi et al. (2007). Contrary, to the bulk of this empirical evidence, Avramov and Chordia (2006) do find large out-of-sample gains. However, they consider individual stocks as opposed to diversified equity portfolios, which favour optimal strategies over simpler ones.

their attention to mean-variance preferences in computing out-of-sample welfare gains, thus overlooking the fact that investors appear to care about both asymmetries and the tails of the wealth distribution, as indicated by the asset pricing literature (Harvey and Siddique, 2000, Dittmar, 2002, Smith, 2007, Guidolin and Timmermann, 2008a). This leaves open the possibility of out-of-sample welfare gains deriving from the impact of predictable higher order moments of stock returns on optimal portfolio composition.

Moreover, there is mounting evidence that non-linear models in general and Markov-switching models in particular may provide a superior fit to the (multivariate) density of stock returns. For instance, Ang and Chen (2002) report that regime switching models replicate the asymmetries in correlations observed in US stock returns better than multivariate GARCH models do. Lettau and Van Nieuwerburgh (2008) suggest that the presence of changing steady state means in the dividend-price ratio may explain why it proves so difficult to predict stock returns out of sample with such a ratio. Guidolin and Nicodano (2009) find that, both in-sample and out-of-sample, regime switching models with a time-varying covariance matrix fare as well as or better than multivariate GARCH models in an international data set of portfolios sorted on the basis of size. Guidolin and Timmermann (2008b) obtain similar evidence for spread portfolios built by sorting on the basis of both size and the book-to-market ratio.

Our paper provides extensive evidence on the recursive out-of-sample performance of optimal portfolio strategies that exploit either the evidence of regime shifts in the distribution of asset returns and/or time the predictable variation in co-skewness and co-kurtosis when applied to three monthly data sets that are commonly used by both academics and practitioners. We analyze ten US industry portfolios, ten book-to-market (BM) international portfolios (along with the world market portfolio), and eight international and emerging market equity portfolios. We find that, at least to a large extent (with some cautions with reference to the international asset menu), modelling the regime switching nature of stock returns is beneficial, while at least one data set can be found in which timing co-skewness and co-kurtosis matters and improves the realized utility from recursive asset allocation. In these cases, we also report that the out-of-sample (OOS) gains exceed those of naive but robust equally-weighted strategies for investors who have one period horizons, as those studied by DeMiguel et al. (2009).

Several papers have already stressed that predicting higher order moments may greatly affect the composition of optimal portfolios (e.g. Ang and Bekaert, 2002, Guidolin and Timmermann, 2008a,b, Guidolin and Nicodano, 2009, and Jondeau and Rockinger, 2012), because investors overweight stocks that increase positive wealth skewness and reduce excess wealth kurtosis relative to mean-variance (MV) portfolios. These papers—with the exception of Ang and Bekaert (2002) and Guidolin and Timmermann (2008a,b)—use however weekly data, which amplify the importance of higher order moments relative the commonly used monthly return series. Moreover, each focusses on one set of equity indices only. Importantly, most of these papers offer little insight on the magnitude and origins of OOS gains and—when they do (as in Jondeau and Rockinger, 2012)—they do not explore the effects of predictability on long-run portfolio performance. A second contribution of our paper consists in analyzing how the investor horizon, which ranges from 1 month to 10 years, affects ex-post gains.

The prevailing linear forecasting methods—such as those investigated in DeMiguel et al. (2009)—describe stock returns as randomly fluctuating around one slowly moving (conditional) mean return and one constant volatility (covariance, in multivariate applications). Our portfolio strategies are instead based on models for returns that allow stock markets to persistently remain in either a bear or a bull regime, i.e., in which both expected returns, volatilities and covariances may abruptly change. If the US stock market is in a bear regime, future returns are expected to fluctuate around a given mean return with a given volatility—unless the stock market moves to a bull regime, which may happen with a positive probability. If that occurs, future returns fluctuate around a higher mean return with a lower volatility—unless the stock market shifts back to the bear regime. In our analysis, this representation fits the return data better than a Gaussian IID representation according to standard statistical tests for all the data sets analyzed.[2] Importantly, the current nature of the market state, bull or bear, is never observable to investors, who instead simply filter the regime out of current and past information on realized stock returns.

A state-dependent, bull and bear return representation also has a number of advantages from the point of view of portfolio management. Given that returns are assumed to be normal conditional on a given regime, our equity portfolios are characterized by the familiar expected return-variance trade-off conditioning on each regime.[3] It is therefore immediate to identify a "defensive" industry as one having a relatively high return in the bear state, compared to other industries. It is also possible to generalize this concept to higher order moments. A truly defensive industry also contributes to increase the skewness of wealth, i.e. it has a relatively low variance in the bear regime, and to reduce wealth kurtosis by displaying relatively low variance in highly volatile bear markets. For instance, North American and Energy stocks appear to be truly defensive portfolios/assets in our data sets and as such they play a key role in the portfolio of investors with plausible preferences.

There are other well known advantages from using such regime-switching representations. First, the data endogenously identify the number of stock market regimes, without the econometrician having to impose them exogenously. Second, it is possible to estimate higher order moments more precisely with a limited amount of observations, because they are a function of the transition probabilities plus conditional means and covariance coefficients, conditional on the regime (see Timmermann, 2000). Thus, forecasting skewness simply requires the estimation of the parameters that characterize bull and bear markets, as opposed to a simpler nonparametric representation that only rely on expanding windows of data. Third, it is possible to nest other, simpler forecasting models as special cases of a general Markov-switching process. Last but not least, portfolio optimization methods that account for systematic skewness and kurtosis in a regime-switching setting are often cumbersome and/or do not allow for consideration of rich asset menus, characterized by a realistic number of assets: this may prevent their use by practitioners.[4] Our paper uses a tractable approach developed by Guidolin and Timmermann (2008a) which is convenient to implement in the presence of non-normalities and large asset menus.

A related literature deals with predicting and timing the volatility of daily data (see Fleming et al., 2001, and references therein), assuming constant expected returns given the short horizon under scrutiny in

[2]This confirms previous specification tests performed by Ang and Chen (2009) and Guidolin and Nicodano (2009), who also extend the comparison of Markov switching models to other non-linear models such as multivariate GARCH, GARCH-in mean models, and VAR-EGARCH.

[3]However, the predictive density of returns at any future horizon $T \geq 1$ is generally not normal and is instead a mixture of normal distributions.

[4]Numerical techniques such as quadrature methods (e.g., Ang and Bekaert, 2001, Lynch, 2001) may not be very precise when the return distributions are not Gaussian as has been often suggested by empirical research. By contrast, Monte Carlo methods (see Guidolin and Timmermann, 2008b) tend to be slow in multivariate applications.

those papers. Here we investigate whether there are economic gains from predicting and timing up to the fourth moment, all of which are likely to vary over a monthly—or longer—horizon. Lambert and Hübner (2013) isolate global levels of co-skewness and co-kurtosis in the US market by defining investable and pure higher-order moment factors, interpreted as the premia for a unit exposure to each type of co-moment risk. They link these premia to the fact that because MV diversification erodes skewness exposure, investors could decide to remain undiversified in order to capture a positive exposure to skewness. Smith (2007) had dealt with a similar framework in which co-skewness preference is time-varying. However in our paper we take a recursive OOS portfolio perspective and, absent specific assumptions on the structure of the pricing kernel as in Lambert and Hübner (2013), we derive forecasts of total co-skewness and co-kurtosis (i.e., including both idiosyncratic and systematic components) as these all matter in optimal portfolio choice.

Finally, two closely related papers are Jondeau and Rockinger (2012) and Martellini and Zieman (2010). Jondeau and Rockinger (2012) study the economic value of higher-moment timing. They compare the magnitude of higher-moment relative to volatility timing by considering two strategies: a 1-month horizon MV strategy, in which investors only exploit their ability to predict volatility; a 1-month higher-moment strategy, in which investors also exploit any ability to also forecast the distribution of returns.[5] In an application to weekly allocation of wealth among the five largest world stock markets under a four-moment approximation to power utility, they find that the MV criterion results in excessive risk-taking and a significant utility loss as compared to a strategy based on higher moments. In our paper we study instead the dynamics of higher order (co-)moments induced by a simple but realistic MS process, consider a range of alternative data sets, and explicitly consider how the results on economic value change as a function of the investment horizon. Martellini and Zieman (2010) have instead researched practical implemention issues of portfolio choice affected by higher-order moments. They show that portfolio selection can be effectively implemented through factor-based and shrinkage estimators. They find that the use of these improved estimators leads to welfare gains in an out-of-sample perspective even for small sample sizes. Even though our paper also focusses on relatively large and realistic equity diversification problems, given our focus on co-skewness and co-kurtosis dynamics driven by Markov regimes, we ask whether and why a difference in economic value may exist between strategies that simply exploit regimes (keeping a MV baseline preference) and those that effectively condition on predictions of co-higher order moments.

The rest of the paper proceeds as follows. We describe our research design and the optimal asset allocation problem in Section 2. Section 3 describes our three alternative data sets and reports estimation results for each of the data sets. We also show that our two-state Markov switching (MS) models have considerable power in capturing the unconditional (co-)skewness and (co-)kurtosis of the equity portfolio returns under examination. Section 4 reports results concerning optimal allocations and interprets the dynamics of portfolio shares over our OOS testing period. Section 5 is the core of the paper, where recursive, realized OOS performance results are presented and commented. Section 6 concludes. Additional material, for instance concerning solution methods for MS asset allocation models when preferences have a moment-based representation, are collected in Appendices A-C.

[5] To this purpose, Jondeau and Rockinger extend the dynamic conditional correlations (DCC) model to a DCC with multivariate skewed t distribution, which allows for both asymmetry and fat tails and in which the parameters driving the shape of the conditional distribution are allowed to vary over time as a function of past shocks.

2. **Research Design: Optimal Asset Allocation Models**

2.1. *Alternative Portfolio Strategies*

We summarize the portfolio strategies examined in this paper in Table 1. The table lists a total of 31 alternative models. There are three dimensions that define a portfolio strategy:

1. The preferences of the investor as captured by the order of a Taylor expansion around a standard power utility function of terminal wealth, with constant coefficient of relative risk aversion γ. In practice, we consider second-, third-, and fourth-order Taylor expansions, with MV indicating simple mean-variance preferences; MVS reflects the preferences of an investor that likes a high expectation and a high and positive skewness of terminal wealth and dislikes the variance of wealth; MVSK indicates that the investor likes a higher expectation and skewness and dislikes the variance and the kurtosis of terminal wealth; under MVK the investor likes the expectation of terminal wealth, dislikes both variance and excess kurtosis, but she does not care for skewness. We consider three values for γ that are typical in the literature, $\gamma = 2$, 5, and 10.

2. The assumed dynamic process for the returns on the assets in the asset menu. Two baseline models are entertained, one featuring no predictability of asset returns and the other implying both short- and long-run predictability of bull and bear type, as captured by a simple MS model. In the no predictability case, the vector process for the asset returns $\mathbf{r}_t$ in the choice menu follows a simple, single-regime ($k = 1$) Gaussian IID process. As we shall see in Section 3, such single-state models are rejected for all of our asset menus. In the MS case, for simplicity, we also use a simple (possibly, too simple) $k = 2$ model in which asset returns are multivariate normal only conditioning on the (bull or bear) market state, but are otherwise characterized by non-normalities, for instance in the form of non-zero skewness and positive excess kurtosis.

3. Whether short-sale constraints are imposed or not, i.e., whether $\omega_{jt} \in [0, 1]$, where ω_{jt} indicates the weight in asset $j = 1, 2, ..., n$, where n is the number of assets in the menu.

In principle, the combination of nine alternative preference models, two possible assumptions for the process of returns, and imposing or not short sale constraints, ought to originate a total of $12 \times 2 \times 2 = 48$ models.[6] However, notice that when $k = 1$, a single-state Gaussian IID model for asset returns always yields a constant zero skewness and an equally constant zero excess kurtosis for returns and hence for terminal wealth. This means that in the $k = 1$ case, there is no material difference between optimal portfolio choice performed under the MV, MVS, MVK, and MVSK models. As a result, the total number of models we entertain is effectively 30. Additionally, for each of the 30 models listed in Table 1, optimal portfolio selection has been performed for a range of possible investment horizons, i.e., 1, 3, 6, 12, 24, 60, and 120 months. However, we see these horizons as applications of each of the 30 strategies to a different time frame, more then as alternative strategies themselves. Table 1 also shows the presence of one celebrated benchmark in the literature, an equally-weighted asset allocation strategy that corresponds to De Miguel et al.'s (2009) $1/n$ strategy. Of course, under a $1/n$ strategy everything else—preferences, the predictability model, the horizon, as well as short-sales—lose relevance.

[6] The 12 derives from the fact that a second-order MV expansion has been applied three times, for $\gamma = 2$, 5, and 10. The same applies to MVS, MVK, and MVSK as well, for a total of 12.

Table 1 also lists the acronyms that we have mapped into each of the 31 models. These have general structure MV[S][K](k, γ)-[c]. For instance, MV(1,5) means that we are considering a model in which the investor has MV preferences, does not believe in the presence of any predictability in stock returns, and she is characterized by a second-order Taylor expansion around $\gamma = 5$. MVS(2,2)-c refers to an investor with mean-variance-skewness preferences obtained as a Taylor expansion of a power utility function with relatively low risk aversion, who plans her portfolio taking bull and bear dynamics into account, and who imposes no short sale constraints on her decisions. Notice that since most of the paper adopts $\gamma = 5$ as a benchmark and simply tracks the effects of $\gamma = 2$ and 10 as a matter of robustness checks, we shall often use a lighter MV[S][K](k)-[c] notation.

Once this set up is in place, the roadmap of the paper is straightforward. After showing rather compelling (as well as typical) evidence that a number of alternative equity asset menus (see details in Table 2) are characterized by bull and bear dynamics in the form of a simple MS model (see Tables 3 and 4), we proceed to compute optimal portfolios (Table 6) and realized ex-post performance (Table 7) for each of the 31 alternative models detailed in Table 1. Interestingly, when bull and bear market states are allowed for, because they imply the existence of predictability (when the underlying Markov regimes are themselves predictable), optimal portfolio strategies can be computed under a number of alternative assumptions on the nature of the current, time t market state (Table 6). First, we have an "average" (better, unconditional) portfolio composition, obtained under the assumption that the investor does not know which state the market is in, but attributes to each of the two regimes its long-run (steady-state) probability. Second, we may compute portfolio weights conditioning on the fact that the investor may perceive the current, time t market state to be either bull or bear.[7] Finally, we can also measure how each portfolio share changes as the probability of being in a bear/bull state is updated by the investor in real time (Figures 2 and 3).

We measure ex-post performance through three different indicators. One is the return-to-volatility ratio (Sharpe ratio), which provides no information on both skewness and kurtosis of wealth. Thus, a portfolio strategy that increases downside risk ranks as well as another that does not, if it has the same Sharpe ratio. This does not happen with the Sortino ratio, which falls when downside volatility increases. The third performance metric is the certainty equivalent of maximum utility, which—in the case of MVS, MVK, and MVSK investors—also captures the higher moments of wealth and appears to be the realized performance criterion that is most consistent with the general framework of portfolio selection entertained in the paper.

2.2. *Investor Preferences*

We study the time t asset allocation problem for an investor with a T-period investment horizon. Suppose that the investor's utility function $U(W_{t+T}; \boldsymbol{\theta})$ only depends on wealth at time $t+T$, W_{t+T}, and its shape is captured through the parameters in $\boldsymbol{\theta}$. The investor maximizes expected utility by choosing among n risky assets whose continuously compounded returns are given by the vector $\mathbf{r}_t \equiv (r_{1t}\ r_{2t}\ ...\ r_{ht})'$. Portfolio weights are collected in the vector $\boldsymbol{\omega}_t \equiv (\omega_{1t}\ \omega_{2t}\ ...\ \omega_{ht})'$. The portfolio selection problem solved by a

[7] Of course, this applies to any possibly configuration of the time t probabilities assigned by the investor to the event "Market is bull" (and conversely, "Market is bear"). For instance, we could compute optimal portfolio weights at any given horizon and for a specific preference model, when the investor assigns a probability of 68% to the market being in a bull state and 32% to a bear state, wherever this probabilities come from. Equivalently, the fact that we exemplify portfolio calculations assuming that the current regime is either bull or bear never implies that the market state is observable as it is realized.

buy-and-hold investor with initial unit wealth is

$$\max_{\boldsymbol{\omega}_t} E_t\left[U(W_{t+T}(\boldsymbol{\omega}_t);\boldsymbol{\theta})\right]$$
$$s.t.\ W_{t+T}(\boldsymbol{\omega}_t) = \boldsymbol{\omega}_t' \exp\left(\mathbf{R}_{t+T}\right), \tag{1}$$

where $\mathbf{R}_{t+T} \equiv \mathbf{r}_{t+1} + \mathbf{r}_{t+2} + ... + \mathbf{r}_{t+T}$ is the vector of continuously compounded portfolio returns over the $T-$period horizon, and portfolio shares sum to 1. Accordingly, $\exp(\mathbf{R}_{t+T})$ is a vector of cumulated portfolio returns. No short sales can be imposed through the constraint $\omega_{jt} \in [0,1]$ for $j = 1, 2, ..., n$.

As in Guidolin and Timmermann (2008a), we approximate the Von-Neumann Morgenstern expected utility function with a function of four moments of the wealth distribution, obtained as a Taylor expansion around some initial wealth level $v_T > 0$ (in principle dependent on the horizon T) of the form

$$\begin{aligned}\hat{E}_t[U^m(W_{t+T};\boldsymbol{\theta})] &= \sum_{q=0}^{m} \kappa_q E_t[(W_{t+T} - v_T)^q] \\ &= \sum_{q=0}^{m} \kappa_q \sum_{q=0}^{q} (-1)^{q-q} v_T^{q-q} \binom{q}{q} E_t\left[\left(\boldsymbol{\omega}_t' \exp\left(\mathbf{R}_{t+T}\right)\right)^q\right]. \end{aligned} \tag{2}$$

with $\kappa_0 > 0$, and κ_q positive (negative) if q is odd (even). This derives from the fact that under non-satiation and risk aversion, marginal utility is positive ($U' > 0$) and decreasing ($U'' < 0$) in wealth. Assuming decreasing absolute risk aversion, we further have $U''' > 0$ (investors prefer positive skew) while, as shown by Kimball (1993), decreasing absolute prudence implies that $U'''' < 0$. For instance, when the Taylor expansion is stopped at $m = 2$, the investor has MV preferences:

$$\begin{aligned}\hat{E}_t[U^2(W_{t+T};\boldsymbol{\theta})] &= \kappa_0 + \kappa_1 E_t[W_{t+T} - v_T] + \kappa_2 E_t[(W_{t+T} - v_T)^2] \\ &= \kappa_0 - \kappa_1 v_T + \kappa_1 E_t[W_{t+T}] + \kappa_2 E_t[(W_{t+T} - E_t[W_{t+T}])^2] + \kappa_2 (v_T - E_t[W_{t+T}])^2 \\ &= \kappa_0' + \kappa_1 E_t[W_{t+T}] + \kappa_2 Var_t[W_{t+T}], \end{aligned} \tag{3}$$

where $\kappa_0' \equiv \kappa_0 - \kappa_1 v_T + \kappa_2 (v_T - E_t[W_{t+T}])^2$, $\kappa_1 > 0$, and $\kappa_2 < 0$.

In our application we use at most $m = 4$ moments in the preference specification. The weights on the first four moments of the wealth distribution are determined to ensure that our results can be compared to those in the existing literature that mostly uses power utility functions. For a given coefficient of relative risk aversion, γ, the power utility function serves as a guide in setting values of $\{\kappa_q\}_{q=0}^m$ in (2). Expanding the powers of $(W_{t+T} - v_T)$ and taking expectations, we obtain the following expression for the four-moment MVSK preference function:

$$\hat{E}_t[U^4(W_{t+T};\gamma)] = \kappa_{0,T}(\gamma) + \kappa_{1,T}(\gamma) E_t[W_{t+T}] + \kappa_{2,T}(\gamma) E_t[W_{t+T}^2] + \kappa_{3,T}(\gamma) E_t[W_{t+T}^3] + \kappa_{4,T}(\gamma) E_t[W_{t+T}^4], \tag{4}$$

where[8]

$$\kappa_{0,T}(\gamma) \equiv v_T^{1-\gamma}\left[(1-\gamma)^{-1} - 1 - \frac{1}{2}\gamma - \frac{1}{6}\gamma(\gamma+1) - \frac{1}{24}\gamma(\gamma+1)(\gamma+2)\right]$$

[8] The notation $\kappa_{n,T}$ makes it explicit that the coefficients of the fourth order Taylor expansion depend on the investment horizon through the coefficient v_T, the point around which the approximation is calculated. We follow standard practice (e.g. Jondeau and Rockinger (2004)) and set the point around which the Taylor series expansion is computed to $v_T = E_t[W_{t+T-1}]$, the expected value of the investor's wealth for a $T-1$ period investment horizon, assumed to have been already computed; moreover $v_1 = 1$.

$$
\begin{aligned}
\kappa_{1,T}(\gamma) &\equiv \frac{1}{6}v_T^{-\gamma}\left[6+6\gamma+3\gamma(\gamma+1)+\gamma(\gamma+1)(\gamma+2)\right] > 0 \\
\kappa_{2,T}(\gamma) &\equiv -\frac{1}{4}\gamma v_T^{-(1+\gamma)}\left[2+2(\gamma+1)+(\gamma+1)(\gamma+2)\right] < 0 \\
\kappa_{3,T}(\gamma) &\equiv \frac{1}{6}\gamma(\gamma+1)(\gamma+3)v_T^{-(2+\gamma)} > 0 \\
\kappa_{4,T}(\gamma) &\equiv -\frac{1}{24}\gamma(\gamma+1)(\gamma+2)v_T^{-(3+\gamma)} < 0.
\end{aligned} \tag{5}
$$

Notice that the expected utility from final wealth increases in $E_t[W_{t+T}]$ and $E_t[W_{t+T}^3]$, so that higher expected returns and more right-skewed distributions lead to higher expected utility. Conversely, expected utility is a decreasing function of the second and fourth moments of the terminal wealth distribution.

In particular, MVS preferences are given by:

$$
\hat{E}_t[U^3(W_{t+T};\gamma)] = \kappa_{0,T}(\gamma) + \kappa_{1,T}(\gamma)E_t[W_{t+T}] + \kappa_{2,T}(\gamma)E_t[W_{t+T}^2] + \kappa_{3,T}(\gamma)E_t[W_{t+T}^3] \tag{6}
$$

where $\kappa_{0,T}(\gamma) \equiv v_T^{1-\gamma}\left[(1-\gamma)^{-1} - 1 - \frac{1}{2}\gamma - \frac{1}{6}\gamma(\gamma+1)\right]$, $\kappa_{1,T}(\gamma) \equiv v_T^{-\gamma}\left[1+\gamma+\frac{1}{2}\gamma(\gamma+1)\right] > 0$, $\kappa_{2,T}(\gamma) \equiv -\frac{1}{2}\gamma v_T^{-(1+\gamma)}(2+\gamma) < 0$, and $\kappa_{3,T}(\gamma) \equiv \frac{1}{6}\gamma(\gamma+1)v_T^{-(2+\gamma)} > 0$. MV preferences simplify to:

$$
\hat{E}_t[U^2(W_{t+T};\gamma)] = \kappa_{0,T}(\gamma) + \kappa_{1,T}(\gamma)E_t[W_{t+T}] + \kappa_{2,T}(\gamma)E_t[W_{t+T}^2] \tag{7}
$$

where $\kappa_{0,T}(\gamma) \equiv v_T^{1-\gamma}\left[(1-\gamma)^{-1} - 1 - \frac{1}{2}\gamma\right]$, $\kappa_{1,T}(\gamma) \equiv v_T^{-\gamma}(1+\gamma) > 0$, and $\kappa_{2,T}(\gamma) \equiv -\frac{1}{2}\gamma v_T^{-(1+\gamma)} < 0$.[9]

2.3. *The Return Process*

We assume that the vector of continuously compounded returns, $\mathbf{r}_t$, is generated by a MS vector autoregressive process driven by a common unobservable state variable, S_t, that takes integer values between 1 and k:

$$
\mathbf{r}_t = \boldsymbol{\mu}_{S_t} + \sum_{i=1}^{p} \mathbf{A}_{i,S_t}\mathbf{r}_{t-i} + \boldsymbol{\varepsilon}_t. \tag{8}
$$

Here $\boldsymbol{\mu}_{S_t} = (\mu_{1S_t}, ..., \mu_{hS_t})'$ is a vector of intercepts in state S_t, $\mathbf{A}_{i,S_t}$ is an $n \times n$ matrix of autoregressive coefficients associated with the ith lag in state s_t, and $\boldsymbol{\varepsilon}_t = (\varepsilon_{1t}, ..., \varepsilon_{ht})' \sim N(\mathbf{0}, \boldsymbol{\Omega}_{S_t})$ is a vector of Gaussian return innovations with zero mean vector and state-dependent covariance matrix $\boldsymbol{\Omega}_{S_t}$:

$$
\boldsymbol{\Omega}_{S_t} = E\left[\left(\mathbf{r}_t - \boldsymbol{\mu}_{S_t} - \sum_{i=1}^{p}\mathbf{A}_{i,S_t}\mathbf{r}_{t-i}\right)\left(\mathbf{r}_t - \boldsymbol{\mu}_{S_t} - \sum_{i=1}^{p}\mathbf{A}_{i,S_t}\mathbf{r}_{t-i}\right)' \Bigg| S_t\right]. \tag{9}
$$

The state-dependence of the covariance matrix captures the possibility of heteroskedastic shocks to asset returns, which is supported by strong empirical evidence, see e.g., Bollerslev et al. (1992). Each state is assumed to be the realization of a first-order, homogeneous Markov chain as the transition probability matrix, $\mathbf{P}$, governing the evolution in the common state variable, S_t, has elements

$$
\mathbf{P}[i,j] = \Pr(S_t = j | S_{t-1} = i) = p_{ij}, \quad i,j = 1,..,k. \tag{10}
$$

Conditional on knowing the state next period, the return distribution is Gaussian. However, since future states are never known in advance, the return distribution is a mixture of normals with the mixture weights reflecting both current state probabilities and transition probabilities.

[9] The MVK case is obtained as the case: $\hat{E}_t[U^4(W_{t+T};\gamma)] = \kappa_{0,T}(\gamma) + \kappa_{1,T}(\gamma)E_t[W_{t+T}] + \kappa_{2,T}(\gamma)E_t[W_{t+T}^2] + \kappa_{4,T}(\gamma)E_t[W_{t+T}^4]$.

Even in the absence of autoregressive terms, (8)-(10) imply time-varying investment opportunities. For example, the conditional mean of asset returns is an average of the vector of mean returns, $\boldsymbol{\mu}_{S_t}$, weighted by the filtered state probabilities $(\Pr(S_t = 1|\mathcal{F}_t),..., \Pr(S_t = k|\mathcal{F}_t))'$, conditional on information available at time t, $\mathcal{F}_t$. Since these state probabilities vary over time, the expected return will also change. When regimes are persistent and mean returns, $\boldsymbol{\mu}_{S_t}$, differ across states, expected returns (more generally, odd-order moments) vary over time. Similarly, when the covariance matrices, $\boldsymbol{\Sigma}_{S_t}$, differ across states, there will be predictability in higher order moments such as volatilities, correlations, skews and tail thickness. Predictability is therefore not confined to mean returns but carries over to the entire return distribution (see, e.g., Ryden et al., 1998).

These properties can easily be visualized in the case of a simple bull and bear model with no vector autoregressive terms ($k = 2$) when $n = 1$:

$$r_t = S_t\mu_{bear} + (1 - S_t)\mu_{bull} + [S_t\sigma_{bear} + (1 - S_t)\sigma_{bull}]\eta_t \quad \eta_t \sim N(0, 1). \tag{11}$$

This means that expected returns will be higher when $S_t = 0$ than when $S_t = 1$, $\mu_{bull} > \mu_{bear}$. While it is trivial to see that $E_{t-1}[r_t] = \pi_{t-1}\mu_{bear}+(1-\pi_{t-1})\mu_{bull}$, because more generally $E_{t-1}\left\{[r_t - E_{t-1}[r_t]]^q\right\} = \pi_{t-1}E\left\{[r_t - E_{t-1}[r_t]]^q|S_t = bear\right\}+(1-\pi_{t-1})E\left\{[r_t - E_{t-1}[r_t]]^q|S_t = bull\right\}$, where $\pi_{t-1} \equiv \Pr(S_t = bear|\mathcal{F}_{t-1})$ it is easy to prove that

$$\begin{aligned}
Var_{t-1}[r_t] &= \pi_{t-1}\sigma^2_{bear} + (1 - \pi_{t-1})\sigma^2_{bull} + \pi_{t-1}(1 - \pi_{t-1})(\mu_{bull} - \mu_{bear})^2 \\
E_{t-1}\left\{[r_t - E_{t-1}[r_t]]^3\right\} &= \pi_{t-1}(1\text{-}\pi_{t-1})(\mu_{bear} - \mu_{bull})^3(1\text{-}2\pi_{t-1}) + 3\pi_{t-1}(1\text{-}\pi_{t-1})(\mu_{bear} - \mu_{bull})(\sigma^2_{bear}\text{-}\sigma^2_{bull}) \\
E_{t-1}\left\{[r_t - E_{t-1}[r_t]]^4\right\} &= 3\pi_{t-1}\sigma^4_{bull} + 3(1\text{-}\pi_{t-1})\sigma^4_{bear} + \pi_{t-1}(1\text{-}\pi_{t-1})(\mu_{bear} - \mu_{bull})^4(1 - 3\pi^2_{t-1} + 3\pi_{t-1}) + \\
&\quad +6\pi_{t-1}(1 - \pi_{t-1})(\mu_{bear} - \mu_{bull})^2(\sigma^2_{bear} - \sigma^2_{bull}).
\end{aligned}$$

Three implications emerge from this simple example. First, the variance of returns is not just a probability-weighted combination of regime-specific variances and in general $Var_{t-1}[r_t] > \pi_{t-1}\sigma^2_{bear} + (1 - \pi_{t-1})\sigma^2_{bull}$ because $\pi_{t-1}(1 - \pi_{t-1})(\mu_{bull} - \mu_{bear})^2 > 0$ by construction. When $\pi_{t-1}(1 - \pi_{t-1})$ is maximum (this occurs when $\pi_{t-1} = 0.5$) and/or $(\mu_{bull} - \mu_{bear})$ is large, then the implied variance from a bull and bear model may considerably exceed variance in each of the two states, including the bear state. Second, because when $(\mu_{bear} - \mu_{bull}) = 0$, independently of the magnitude of $(\sigma^2_{bear} - \sigma^2_{bull})$, we have that $E_{t-1}\left\{[r_t - E_{t-1}[r_t]]^3\right\} = 0$, this means that the presence of regime switching in means is a necessary condition for non-zero skewness to obtain. Third, although neither $(\sigma^2_{bear} - \sigma^2_{bull}) = 0$ nor $(\mu_{bear} - \mu_{bull}) = 0$ does not imply that $E_{t-1}\left\{[r_t - E_{t-1}[r_t]]^4\right\} = 0$, both $\mu_{bear} \neq \mu_{bull}$ and $\sigma^2_{bear} \neq \sigma^2_{bull}$ contribute to increase $E_{t-1}\left\{[r_t - E_{t-1}[r_t]]^4\right\}$ to exceed $3\pi_{t-1}\sigma^4_{bull} + 3(1 - \pi_{t-1})\sigma^4_{bear}$, so that regimes in both expected returns and in their variance do inflate the tails of the predicted density of returns.[10]

2.4. *Approximate Solution Methods*

We now indicate how to solve the investor's optimal asset allocation problem when preferences are defined over moments of terminal wealth (2) and returns follow the regime switching process (8)-(10).[11] Start by

[10] These claims depend on the fact that $1 - 3\pi^2_{t-1} + 3\pi_{t-1} > 0$ for any $\pi_{t-1} \in [0, 1]$ and use the definitions of scaled skewness, $Skew_{t-1}[r_t] = E_{t-1}\left\{[r_t - E_{t-1}[r_t]]^3\right\} / [Var_{t-1}[r_t]]^{3/2}$, and of scaled kurtosis, $Kurt_{t-1}[r_t] = E_{t-1}\left\{[r_t - E_{t-1}[r_t]]^4\right\} / [Var_{t-1}[r_t]]^2$,

[11] Alternative solution methods to (1) under predictability of returns are described in Ang and Bekaert (2002), Barberis (2000), Brandt (1999), Campbell, Chan and Viceira (2003), Kandel and Stambaugh (1996), and Lynch (2001).

noting that the qth moment of the cumulated return on the portfolio is given by:

$$E_t\left[\left(\boldsymbol{\omega}_t' \exp\left(\mathbf{R}_{t+T}\right)\right)^q\right] = \sum_{q_1=0}^{q} \cdots \sum_{q_n=0}^{q} \lambda(q_1, q_2, ..., q_n) \left(\prod_{i=1}^{n} \omega_i^{q_i}\right) M_{t+T}^{(q)}(q_1, ..., q_n), \tag{12}$$

where $\sum_{i=1}^{n} q_i = q$, $0 \leq q_i \leq q$ $(i = 1, ..., n)$,

$$\lambda(q_1, q_2, ..., q_n) \equiv \frac{q!}{q_1!q_2! \ldots q_n!}. \tag{13}$$

and $M_{t+T}^{(q)}(q_1, ..., q_n)$ can be evaluated recursively, using equations reported in Appendix A. The moments of the wealth distribution can thus be obtained by solving a small set of difference equations corresponding to the number of regimes. The otherwise complicated numerical problem of optimal asset allocation is then reduced to one of solving for the roots of a low-order polynomial. This solution is closed-form in the sense that it is computable with a finite number of elementary operations. In practice, the task consists of solving a system of cubic equations in $\hat{\boldsymbol{\omega}}_t$ derived from the first and second order conditions

$$\nabla_{\boldsymbol{\omega}_t} \hat{E}_t[U^4(W_{t+T};\gamma)]\Big|_{\hat{\boldsymbol{\omega}}_t} = \mathbf{0}', \quad H_{\boldsymbol{\omega}_t} \hat{E}_t[U^4(W_{t+T};\gamma)]\Big|_{\hat{\boldsymbol{\omega}}_t} \text{ is negative definite.} \tag{14}$$

Thus $\hat{\boldsymbol{\omega}}_t$ sets the gradient, $\nabla_{\boldsymbol{\omega}_t} \hat{E}_t[U^4(W_{t+T};\gamma)]$, to a vector of zeros and produces a negative definite Hessian matrix, $H_{\boldsymbol{\omega}_t} \hat{E}_t[U^4(W_{t+T};\gamma)]$.

Putting together the preferences in Section 2.2 and the multivariate return process in Section 2.3 (with $k = 2$), the resulting optimal portfolio composition—which in a standard MV problem with Gaussian return depends only on the vector of expected returns, the covariance matrix of returns, and on risk aversion—depends on:

1. Differences between mean returns, μ_1 and μ_2, and variances, σ_1, σ_2, (and more generally covariance parameters) across states.
2. The current state probabilities $(\pi_t,\, 1 - \pi_t)'$ which determine moments of returns at all future points provided that either the mean or (co-)variance parameters differ across states ($\mu_1 \neq \mu_2$ or $\sigma_1 \neq \sigma_2$).
3. State transition probabilities which also affect the speed of mean reversion in the investment opportunity set towards its steady state.
4. The number of moments of the wealth distribution that matters for preferences, m, in addition to the weights on the various moments.
5. The investment horizon, T.

In what follows, our benchmark results assume that $\gamma = 5$, even though we later present robustness checks that allow this coefficient to assume both larger and smaller values.

3. **Empirical Results**

3.1. *Data and Descriptive Statistics*

We analyze recursive realized performance for three relatively large data sets of monthly equity returns. The first one—which we label IE (from international equities)—comprises eight Morgan Stanley Capital

International MSCI) international value-weighted portfolios for macro-regions, five concerning developed markets (Europe ex-United Kingdom, Japan, North America, Pacific ex-Japan, and United Kingdom) and three concerning instead emerging markets (EM Latin America, EM Asia, and EM Europe & Middle-East). The sample period of our analysis is 1988:01-2008:08. The second data set covers ten US value-weighted equity industry portfolios (we call it IND), from July 1926 to July 2008, obtained from CRSP through Kenneth French's data library.[12] The portfolios are Nondurables, Durables, Manufacturing, Energy, High Tech, Telecommunications, Shops/Distribution, Health, Utilities, and a residual "Other" category. The third data set consists of ten Book-to-Market (we call it BMINT) sorted portfolios from five geographical (international) areas plus the MSCI World market portfolio over the sample period 1975:01-2007:12. The ten combinations of geographical areas and BM sorting consists of EU ex-UK ex-Scandinavia Value, EU ex-UK ex-Scandinavia Growth, United Kingdom Value, United Kingdom Growth, Asia & Pacific Value, Asia & Pacific Growth, Scandinavia Value, Scandinavia Growth, United States Value, United States Growth. In this case, there is an eleventh portfolio which is represented by the MSCI value-weighted world portfolio.[13] Also the BMINT data set is obtained from Kenneth French's data library.[14] Importantly, all our data sets—and significantly, IE and BMINT—feature total returns (i.e., inclusive of dividends) denominated in dollars. This implies that our perspective is that of a US investor who is completely un-hedged as typical of many other papers on international portfolio diversification (e.g., Ang and Bekaert, 2002).[15]

Our choice of the IE, IND, and BMINT data sets, if anything, distorts our results against a finding of large non-normalities. Indeed, our focus is neither on individual security returns nor on data sampled at higher frequencies, where it is easier—for economic reasons in the case of individual stock returns that contain substantial idiosyncratic components, for statistical reasons in the case of high-frequency returns that contain massive amounts of noise—to uncover non-normal features. We also avoid analyzing size and momentum portfolios, let alone hedge-fund returns, that are already known to display asymmetries that are exploitable in a portfolio setting (see, e.g., Guidolin and Nicodano, 2008; Hong et al., 2007). Each of the data sets is supposed to contribute to our analysis in a distinct and important dimension. IND is a very long data set with relevance to the domestic portfolio diversification of a US investor that should allow us to identify a large number of bull and bear spells. IE is the typical data that has been employed in the literature on international diversification, enriched to include high-quality MSCI index returns for emerging markets, which are the markets most likely to propose substantial non-normalities and time variation in them. BMINT keeps an international focus but also reflects asset pricing concepts—such as the possibly different pricing of values vs. growth stocks—that have been shown to follow multi-state, regime switching

[12]See http://mba.tuck.dartmouth.edu/pages/faculty/ ken.french/data_library.html

[13]This is similar to the portfolio allocation approach in papers such as Guidolin and Timmermann (2008a,b) or Lynch (2002) in which the (I)CAPM benchmark—such as a domestic or world market portfolio—is part of the asset menu to allow one to test the portfolio restriction of the (I)CAPM that asset demands ought to be composed of a 100% allocation to the market portfolio and nothing to the remaining portfolio. Although logically the presence of the market portfolio in the asset menu poses no issues, it means that in practice, mapping optimal portfolio compositions in specific trades on the stocks that compose the different portfolios will require an extra layer of calculations that we do not pursue in this paper.

[14]The raw data are from Morgan Stanley Capital International for 1975 to 2006 and for 2007 from Bloomberg. Firms in the country portfolios are value-weighted. To construct index returns, the weight of each country is equale to its EAFE weight.

[15]The reason is of course the optimal hedging choice for any given position in international equities should be modelled and will come to depend on both the investor's preferences as well as the predictability model she adopts. Therefore, considering 100% fully hedged equity portfolio returns would be even more arbitrary and less natural than investigating international asset allocation decisions before any hedging decisions are made.

dynamics of primary importance in an asset allocation perspective (see e.g., Guidolin and Timmermann, 2008b); the relatively long, 35-year sample covered by BMINT allows us to capture the important bear spells of the world equity markets between the mid-1970s and early 1980s.

All of our data sets are truncated to end before the Lehman Brothers' bankruptcy in September 2008 to avoid that our results may be dominated by the large regime shifts (to a deep, crisis bear state) and ensuing large non-normal features that have manifested themselves between late 2008 and 2009. For instance, Guidolin and Ria (2011) have shown that when MS models are extended to model U.S. data from the 2008-2009 period, usually realized performance results under MS considerably improve. Intuitively, this simply derives from the ability of MS models to handle the presence of multiple regimes. However, because our paper is not really or mostly about the realized, OOS portfolio effects of taking regime into account, but instead concerns the economic value of higher order (co-)moments, it seems optimal to exclude data for the most recent, 2009-2012 period.

Table 2 reports summary statistics, with the lower portions displaying the correlation/co-kurtosis matrix and the co-skewness matrix, respectively. The table is organized in three distinct panels, A-C, devoted to the IE, IND, and BMINT data sets, respectively. Panel A shows a wide dispersion of monthly mean returns and volatilities across international equity portfolios, although not all the mean estimates for IE returns are statistically significant, as one would expect. Also the ratio of expected return to volatility (Sharpe) spans a wide range, from -0.064 and 0.059 for Japan and EM Asia, to 0.193 and 0.174, respectively for EM Europe & Middle East and EM Latin America.[16] Correlations involving the UK and North America are generally higher than those involving Emerging Markets, whose cross-correlations never exceed a moderate 0.5. Panel A of the Table also shows that five of the returns series under examination either display a statistically significant negative skewness (Europe ex-UK and North America), or a statistically significant excess kurtosis (EM Europe & Middle East), or both (Pacific ex-Japan and EM Latin America). As a result, for these five indices, a Jarque-Bera test rejects the null of normality. Interestingly, the rejection also concerns EM Asia even though neither skewness nor excess kurtosis were individually statistically significant. However, given the fact that a Jarque-Bera test is structured as a joint test of non-zero skewness and excess kurtosis, this is not surprising. It is similarly not surprising our finding that non-normalities are pervasive in the case of EM index returns, although as we have seen this result also extends to continental European, North-American, and Pacific equity portfolios. The last two columns show that while returns are generally not serially correlated in levels, they often are in their squares, which is a common indication of volatility clustering and conditional heteroskedasticity

On the contrary, panel B of Table 2 shows that monthly mean returns are not particularly dispersed in the US IND data set, ranging from 0.83% for Telecommunication to 1.10% for Energy stocks. Strikingly, all these mean return estimates are now highly statistically significant, even though, in comparison to panel A, we are using a considerably longer sample. Yet, because of marked heterogeneity in realized volatilities, their Sharpe ratios differ markedly, from 0.095 for Other Industries to 0.143 for Nondurables. Not surprisingly, industry portfolios display higher cross-correlations than international stocks returns do,

[16] In this Table and in the rest of the paper, Sharpe ratios (unconditional and full-sample as well conditional) are always computed by subtracting from realized returns the appropriate return over an identical sample period of a portfolio that simply invests in US 1-month T-bills. Of course, for an horizon of 1-month, such a portfolio is completely riskless. The choice of using 1-month T-bill returns is consistent with our focus on the diversification decisions of an un-hedged US investor.

always in excess of 0.5 and with peaks in excess of 0.85, when diversification opportunities would be modest in a simple static MV framework. Interestingly, all industry portfolios display no skewness but are characterized by massive excess kurtosis, highly statistically significant in ten cases out of ten. As a result, the Jarque-Bera test uniformly rejects the null of normally distributed returns with p-values that are always below 1%. Moreover, industry returns all present interesting dynamics patterns, as they are both serially correlated in levels and squares.[17]

Dispersion in mean returns and Sharpe ratios appears to be high also in Panel C, devoted to the BMINT data set. As in Fama and French (1998), higher mean returns on value portfolios are the norm, with the exception of the US market, and a peak of 1.73% per month for the UK value portfolio (which exceeds the 1.34% of UK growth). Value portfolios usually yield the highest Sharpe ratios as well, e.g., 0.201 for United Kingdom Value. There are large and significant correlations between Value and Growth portfolios within each geographical region. For instance, the correlation between EU ex-UK ex-Scandinavia Value and EU ex-UK ex-Scandinavia Growth is 0.85. Correlations are more modest when portfolios of different regions are compared and are occasionally below 0.5, even though with 33 years of monthly data it is easy to see that all pair-wise correlations are significantly positive. While only EU ex-UK ex-Scandinavia Value has a significantly negative skewness coefficient, the finding of a statistically significant excess kurtosis is much more widespread and concerns at least six portfolios. As a result (and even in the four cases when neither skewness nor excess kurtosis are significant), the Jarque-Bera test always rejects the null of normality. Also in the case of BMINT, for a majority of the return series there widespread evidence of volatility clustering.

3.2. *Sample Higher Co-Moments*

An investor with higher-order preferences cares about the higher order moments of terminal wealth and therefore about co-moments in equity returns. For instance, the objective function in (4) features the terms $E_t[W_{t+T}^2]$, $E_t[W_{t+T}^3]$ and $E_t[W_{t+T}^4]$. For concreteness, consider the simple case of $T = 1$ and only two stocks, $n = 2$, to which the investor commits the weights ω_{1t} and $(1 - \omega_{1t})$. Because $W_{t+1} = \omega_{1t}(1 + r_{1t+1}) + (1 - \omega_{1t})(1 + r_{2t+1})$,

$$\begin{aligned}
E_t[W_{t+T}^2] &= \omega_{1t}^2 E_t[(1 + r_{1t+1})^2] + (1 - \omega_{1t})^2 E_t[(1 + r_{2t+1})^2] + 2\omega_{1t}(1 - \omega_{1t})E_t[(1 + r_{1t+1})(1 + r_{2t+1})] \\
E_t[W_{t+T}^3] &= \omega_{1t}^3 E_t[(1 + r_{1t+1})^3] + (1 - \omega_{1t})^3 E_t[(1 + r_{2t+1})^3] + 3\omega_{1t}^2(1 - \omega_{1t})E_t[(1 + r_{1t+1})^2(1 + r_{2t+1})] + \\
&\quad +3\omega_{1t}(1 - \omega_{1t})^2 E_t[(1 + r_{1t+1})(1 + r_{2t+1})^2] \\
E_t[W_{t+T}^4] &= \omega_{1t}^4 E_t[(1 + r_{1t+1})^4] + (1 - \omega_{1t})^4 E_t[(1 + r_{2t+1})^4] + 4\omega_{1t}^3(1 - \omega_{1t})E_t[(1 + r_{1t+1})^3(1 + r_{2t+1})] + \\
&\quad +6\omega_{1t}^2(1 - \omega_{1t})^2 E_t[(1 + r_{1t+1})^2(1 + r_{2t+1})^2] + 4\omega_{1t}(1 - \omega_{1t})^3 E_t[(1 + r_{1t+1})(1 + r_{2t+1})^3]. \quad (15)
\end{aligned}$$

Although these expressions cannot be directly mapped into (2) because we work with continuously compounded returns, they highlight a few interesting points. $E_t[W_{t+T}^2]$ is a function of the second non-central moments of gross returns, each weighted by the square of the associated portfolio weights plus twice the weighted first cross moment, $E_t[(1 + r_{1t+1})(1 + r_{2t+1})]$ which is obviously related to covariance by $E_t[(1+r_{1t+1})(1+r_{2t+1})] = Cov_t[(1+r_{1t+1}), (1+r_{2t+1})] + E_t[1+r_{1t+1}]E_t[1+r_{2t+1}]$. $E_t[W_{t+T}^3]$ is a function

[17] While the presence of structure in squares is hardly surprising, the presence of modest degrees of predictability in the returns themselves may possibly depend on our use of a very long sample that includes data from the 1920s and 1930s, when it is more typical to find slowly mean reverting components in (monthly) price series.

of the third non-central moments of gross returns, each weighted by the third power of portfolio weights, plus three times the weighted cross moments $E_t[(1+r_{1t+1})^2(1+r_{2t+1})]$ and $E_t[(1+r_{1t+1})(1+r_{2t+1})^2]$ which are obviously related to special covariance terms. For instance, $E_t[(1+r_{1t+1})^2(1+r_{2t+1})] = Cov_t[(1+r_{1t+1})^2, (1+r_{2t+1})] + E_t[(1+r_{1t+1})^2]E_t[1+r_{2t+1}]$, which is an adjusted covariance between the squares of gross stock returns on stock 1 and the level of stock returns of stock 2.[18] In this paper, we refer to quantities such as $E_t[(1+r_{lt+1})^2(1+r_{qt+1})]$ as *co-skewness coefficients*. Interestingly, the conditional non-central third moment of time $t+1$ wealth does not only depend on a weighted sum of the conditional non-central third moments of the two stocks, but also on a weighted sum of co-skewness coefficients involving the two assets in the menu. Finally, $E_t[W_{t+T}^4]$ is a function of the fourth non-central moments of gross returns, each weighted by the fourth power of portfolio weights, plus four times the weighted cross moments $E_t[(1+r_{1t+1})^3(1+r_{2t+1})]$ and $E_t[(1+r_{1t+1})(1+r_{2t+1})^3]$, which can be interpreted as an adjusted covariance between the third power of gross stock returns on a stock and the level of returns on the other stock. Additionally, in this case also six times the weighted conditional expectation of squared stock returns, $E_t[(1+r_{1t+1})^2(1+r_{2t+1})^2] = Cov_t[(1+r_{1t+1})^2, (1+r_{2t+1})^2] + E_t[(1+r_{1t+1})^2]E_t[(1+r_{1t+1})^2]$, which is a covariance between squares of gross stock returns adjusted by the product of conditional non-central second moments on each of the stocks. In this paper, we refer to quantities such as $E_t[(1+r_{lt+1})^2(1+r_{qt+1})^2]$ and $E_t[(1+r_{lt+1})^3(1+r_{qt+1})]$ as *co-kurtosis coefficients*. Interestingly, the conditional non-central fourth moment of time $t+1$ wealth does not only depend on a weighted sum of the conditional non-central fourth moments of the two stocks, but also on a weighted sum of co-kurtosis coefficients.

More generally, we measure the scaled co-skewness of a triplet of stock returns $i, j, l = 1, ..., n$ as in Jondeau and Rockinger (2006):

$$S_{i,j,l} \equiv \frac{E[(r_{it} - E[r_{it}])(r_{jt} - E[r_{jt}])(r_{lt} - E[r_{lt}])]}{\{E[(r_{it} - E[r_{it}])^2]E[(r_{jt} - E[r_{jt}])^2]E[(r_{lt} - E[r_{lt}])^2]\}^{1/2}}. \tag{16}$$

When $i = j = l$, $S_{i,j,l}$ reduces to the third central moment of returns on asset i, which captures the traditional measure of scaled skewness, $Skew_i = S_{i,i,i}/\sigma_i^3$ reported in the upper portions of Table 2. When $i \neq j \neq l$, $S_{i,j,l}$ gives a signed measure of the strength of the linear association among deviations of returns from their means across triplets of asset returns. A risk-averse investor dislikes negative values of $S_{i,j,l}$ corresponding to cases when returns in different markets are below their mean at the same time. When only the returns on two assets are involved, $S_{i,j,j}$ reflects the strength of the linear association between squared deviations from the mean and signed deviations from the mean for a pair of assets. A security i with negative $S_{-i,i,i}$ coefficients for the majority of all possible pairs of returns on other securities (denoted as $-i$) is a security that becomes highly volatile when other securities give low returns, and vice-versa. To a risk averse investor this is an unattractive feature since risk rises in periods with low returns. A security i with predominantly negative $S_{i,-i,-i}$ coefficients pays low (high) returns when other securities become highly volatile; again this feature is harmful to a risk-averse investor since the security performs poorly when other assets are highly risky. The bottom sections of the various panels of Table 2 report the estimated sample $S_{-i,i,i}$ and $S_{i,-i,-i}$ respectively above and below the diagonal.[19]

[18] Although it is clear that $Cov[E_t[(1+r_{1t+1})^2], (1+r_{2t+1})]$ does not correspond to $Cov_t[(1+r_{1t+1})^2, (1+r_{2t+1})]$ unless special assumptions are made, it is common to interpret objects such as $Cov_t[(1+r_{1t+1})^2, (1+r_{2t+1})]$ as a covariance between the variance of returns on stock 1 and the level of stock 1 returns.

[19] We have also computed $S_{i,j,l}$ when $i \neq j \neq l$, but we omit reporting these sample co-skewness coefficients because this

Turning to fat tails in the return distribution, the scaled co-kurtosis of a set of four stock returns $i, j, l, q = 1, ..., n$ is equal to:

$$K_{i,j,l,q} \equiv \frac{E[(r_{it} - E[r_{it}])(r_{jt} - E[r_{jt}])(r_{lt} - E[r_{lt}])(r_{qt} - E[r_{qt}])]}{\{E[(r_{it} - E[r_{it}])^2]E[(r_{jt} - E[r_{jt}])^2]E[(r_{lt} - E[r_{lt}])^2]E[(r_{qt} - E[r_{qt}])^2]\}^{1/2}}. \qquad (17)$$

When $i = j = l = q$, $K_{i,j,l,q}$ becomes proportional to the coefficient of kurtosis, $Kurt_i = K_{i,i,i,i}/\sigma_i^4$ reported in the upper portions of Table 2. When $i \neq j \neq l \neq q$, $K_{i,j,l,q}$ gives a signed measure of the strength of the linear association among deviations of returns from their means across four-tuples of asset returns. The term $K_{i,i,j,j}$, which is displayed in the middle portion of panels A-C of Table 2, sheds light on the correlation between volatility shocks across markets. Large positive values are undesirable, reflecting that volatility tends to be large at the same time in market i as in other market, thus increasing the overall portfolio risk. $K_{i,i,i,-i}$ measures the signed linear association between cubic and level deviations from means for a pair of assets. A security i with positive values of $K_{i,i,i,-i}$ becomes skewed to the left when other securities pay below-normal returns and is hence undesirable to risk-averse investors.[20]

Table 2, panels A-C, provide additional information by computing sample higher (unconditional, central) co-moments of the type $S_{i,i,j}$ and $K_{i,i,j,j}$ ($i \neq j$) which simply involve pairs of stock portfolios.[21] From the upper portion of Table 2 it is clear that the Jarque-Bera test rejects normality in five portfolios out of eight in panel A (IE data set), and always (i.e., in 20 cases out of 20) in panels B and C. This is relevant because the Jarque-Bera test is based on a weighted combination of the scaled coefficients $S_{i,i,i}$ and $K_{i,i,i,i}$ ($i = 1,..., n$). As we have seen, in many cases $S_{i,i,i}$ and/or $K_{i,i,i,i}$ turned out to be statistically significant. In panel A, we also find that sample scaled co-kurtosis $K_{i,i,j,j}$ ($i \neq j = 1,..., n$) rarely exceeds the multivariate Gaussian benchmark, i.e., there is weak evidence of excess co-kurtosis of the $K_{i,i,j,j}$ type. The only significant cases of non-normal volatility contagion across regions concern the pairs UK/North America, Pacific ex-Japan/EM Asia, and UK/Europe ex-UK, and UK/EM Latin America. As far as, the scaled co-skewness coefficients of type $S_{i,i,j}$ ($i \neq j = 1,..., n$) are concerned, out of 56 possible coefficients, we obtain that only 13 are statistically significant (notice that under multivariate normality, all co-skewness coefficients have to be zero).[22] 10 such cases involve two specific portfolios, Pacific ex-Japan and Europe ex-UK, with values that are statistically significant and negative. All in all, although there is some evidence of non-normality in IE data, such evidence is not extremely strong and weaker than what has been reported by other papers on different sample periods and data (e.g., 1974-2004 in Guidolin and Timmermann, 2008a).

The empirical evidence of rich co-skewness and co-kurtosis patterns is considerably stronger in panels B and C, for data sets IND and BMINT. However, also in this case a few features are remarkable. In the case of the US industry data set, while *all* (which is indeed striking) $K_{i,i,j,j}$ co-skewness coefficients

would be extremely unpractical due to space constraints. However, our solution methodology will reflect the values implied by our MS model for $S_{i,j,l}$ (or related, conditional quantities) in all permutations of the indices.

[20] Also in this case, we have computed $K_{i,i,i,j}$ for all possible indices j, but we omit reporting these sample co-kurtosis coefficients to save space. However, our solution methodology will reflect the values implied by our MS model for $K_{i,j,l,q}$ (or related, conditional quantities) for all permutations of the indices.

[21] We limit our analysis to pairs of stocks only for reasons of space, although it is clear that with $h > 2$ assets as everywhere in this paper, many more scaled co-skewness and co-kurtosis coefficients have been computed.

[22] In the co-skewness matrix, coefficients above the main diagonal refer to the sample covariance between the square of the returns of the row portfolio and the level of returns of the column portfolio; coefficients below the main diagonal refer to the sample covariance between the level of the returns of the column portfolio/index and the square of returns of the row portfolio/index.

are statistically in excess of the underlying normal counterparts—i.e., there is massive evidence of excess co-movement in return volatilities—there is no evidence of any statistically or even economically relevant excess co-skewness of the $S_{i,i,j}$ type. In this sense, IND can be taken to represent a case in which MS "bear & bull" models need only to fit co-movements in excess tail thickness and it will be interesting to map the ability to track and forecast such movements into portfolio weights and realized portfolio performance.

Finally, panel C of Table 2 shows that while excess co-kurtosis of the $K_{i,i,j,j}$ type is again always statistically significant in the case of the BMINT data set, now we have that also 24 (out of a total of 111 non-empty cells) co-skewness coefficients of the type $S_{i,i,j}$ are statistically significant, especially those involving squared returns on the MSCI world market portfolio, EU ex-Scandinavia, ex-UK value, and United States Value. All in all, BMINT can be taken as a case in which non-normalities are substantial and widespread, and concern both co-movements in variance and co-movements between the asymmetric shape of the return distribution and the level of returns. In a sense, BMINT provides a case in which all the possible ways in which non-normalities may manifest themselves are in play.

3.3. *Estimation Results*

Empirical analyses destined to provide inputs to portfolio optimization problems often pre-specify an exogenous distribution for returns which is assumed to be correctly specified in statistical terms and to be economically relevant. We instead adopt an agnostic approach and perform several specification tests, allowing our data to endogenously determine whether or not there is evidence of regimes (when $k = 2$, bull and bear markets) in our equity return data. Importantly, we do not limit ourselves to pit against each other the cases of $k = 1$ (which is then a Gaussian VAR process) and $k = 2$, but extend our specification search to the number of regimes k. Table 3 reports the results of these tests, for up to $k = 4$ regimes. We also let the test determine whether there should be a (vector) autoregressive term of order $p = 1$, as opposed to no lags ($p = 0$). In Table 3, the acronym MSIA(1,1) simply refers to a Gaussian VAR(1)—indicating the presence of simple linear predictability in returns, by which past returns may forecast subsequent returns:

$$\mathbf{r}_t = \boldsymbol{\mu} + \mathbf{A}\mathbf{r}_{t-1} + \boldsymbol{\varepsilon}_t \quad \boldsymbol{\varepsilon}_t \sim N(\mathbf{0}, \boldsymbol{\Omega}). \tag{18}$$

MSIA(1,0) refers instead a multivariate Gaussian IID model with unpredictable returns, $\mathbf{r}_t = \boldsymbol{\mu} + \boldsymbol{\varepsilon}_t \sim N(\boldsymbol{\mu}, \boldsymbol{\Omega})$. MSIA($k$,1) allows for a (vector) autoregressive term of order 1 as well as for k regimes, while MSIAH(k,1) adds heteroskedasticity, in the form of a MS covariance matrix. Finally, models of the type MSI(k,0) simply contain MS in the intercept, while MSIH(k,0) models also add a k-state covariance matrix.

The MS model in (8) is estimated by maximum likelihood. Estimation and inferences are based on the EM (Expectation-Maximization) algorithm proposed by Dempster et al. (1977) and Hamilton (1993), a filter that allows the iterative calculation of the one-step ahead forecast of the state vector given the information set $\mathcal{F}_t$ and the consequent construction of the log-likelihood function of the data. Under standard regularity conditions (such as identifiability, stability and the fact that the true parameter vector does not fall on the boundaries), the ML estimator is consistent and asymptotic normal, where the covariance matrix of the parameter estimates is given by the inverse of the (asymptotic) information matrix. As a consequence, and with one important exception, standard inferential procedures are available to test statistical hypothesis. In particular, in this paper, the right-most column of Table 3 will make use of the fact that—if we define $\boldsymbol{\delta}$ the $z \times 1$ vector of unknown MS parameters to be estimated and $r = rank(\partial\phi(\boldsymbol{\delta})/\partial\boldsymbol{\delta}')$—the null hypothesis H_0: $\phi(\boldsymbol{\delta}) = \mathbf{0}$ vs. $H_1 : \phi(\boldsymbol{\delta}) \neq \mathbf{0}$ under the assumption that under both hypothesis the number of regimes k is identical, can be tested using the fact that

$$LR \equiv 2\left[\ln L(\hat{\boldsymbol{\delta}}) - \ln L(\tilde{\boldsymbol{\delta}})|_{\phi(\tilde{\boldsymbol{\delta}})=0}\right] \xrightarrow{d} \chi^2_r, \tag{19}$$

where $\hat{\boldsymbol{\delta}}$ is the unconstrained ML estimator and $\tilde{\boldsymbol{\delta}}$ is the estimator obtained under r restrictions $\phi(\tilde{\boldsymbol{\delta}}) = 0$.

The exception to these standard inferential procedures concerns the number of non-zero rows of the transition matrix $\mathbf{P}$, i.e., the number of regimes, k. In this case, even under the assumption of asymptotic normality of the estimator $\hat{\boldsymbol{\delta}}$, standard testing procedures suffer from non-standard asymptotic distributions of the likelihood ratio test statistic because under any number of regimes smaller than k there are a few structural parameters of the unrestricted model — the elements of the transition probability matrix associated with the rows that correspond to "disappearing states"—that can take any values without influencing the likelihood function. We say that these parameters become a *nuisance* to estimation. We follow a number of papers (see e.g., Guidolin and Timmermann, 2008b), and perform data-driven model selection that relies on information criteria, such the Schwartz, Hannan-Quinn, and Akaike criteria.[23]

In Table 3, the information criteria have a hard time discriminating among alternative return processes in the case of IE data (Panel A). In panel A, we have boldfaced the three models that produce the three best (lowest) values of the three information criteria. In fact, under the parsimonious BIC, it is not clear whether there is a need to model bull and bear markets, or at least a very "minimal" MSI(2,0) in which only expected returns are subject to regime shifts may be selected. This is a sensible finding given that non-normalities were moderate in the case of the IE data set. However, the Hannan-Quinn and AIC criteria favor richer, MS models, including relatively large four-state models. Additionally, a Davies-adjusted LR test of the null of $k = 1$ against $k > 1$ always returns a zero p-value, indicating that the null of a single regime is always strongly rejected. In the light of these conflicting indications, we settle on a relatively parsimonious MSIH(2,0) bull and bear framework with regime-dependent covariance matrices.[24]

In panels B and C, all information criteria give compelling indications in favor of $k \geq 2$, even though not always the criteria agree on which model ought to be selected. In the case of the IND data set in panel B, a two-state bull and bear model MSIH(2,0) minimizes the BIC and yields a Hannan-Quinn among the three lowest criteria. However, the AIC criterion points in the direction of considerably larger three- and four-state models. All in all, also in the light of the dominant role played in the literature by simpler two-regime models (see e.g., Ang and Bekaert, 2002) we settle on a parsimonious MSIH(2,0) model that gives us 75 observations per estimable parameter. The outcomes of the specification search are very similar when applied to the BMINT data set in panel C: the BIC and Hannan-Quinn criteria give indications in favor of a parsimonious bull and bear MSIH(2,0) model with a saturation ratio of almost 28.[25]

[23] Alternative inferential procedures concerning the number of regimes in MS models have been proposed. For instance, Davies (1977) bounds the LR test but avoids the problem of estimating the nuisance parameters and derives instead an upper bound for the significance level of the LR test under nuisance parameters:

$$\Pr(LR > x) \leq \Pr(\chi_1^2 > x) + \sqrt{2x}\exp\left(-\frac{x}{2}\right)\left[\Gamma\left(\frac{1}{2}\right)\right]^{-1}.$$

The bound holds if the likelihood has a single peak. Table 3 shows p-values computed using Davies' bound.

[24] This conclusion was also influenced by the findings in panels B and C of Table 3, where the MSIH(2,0) turns out to be always the best in terms of information critiera. In panel A, the model implies 90 parameters which means that with 1,984 available observations this makes available approximately 22 observations per parameter to perform MLE estimation.

[25] These choices are consistent with Catao and Timmermann (2007), who construct pure country and pure industry factor

We can now turn to Table 4, which displays estimates of the parameters of the multivariate Gaussian IID return process in top portion of the table and of the two-regime model in the lower portion. In our description below, we focus on the bull and bear predictability framework, as the parameter estimated of the single-state, no predictability model mirror closely the sample descriptive statistics already commented with reference to Table 2. In one state—which we name Bear regime—most equity portfolios have relatively low mean returns. In fact, in the bear state, for some portfolios the estimates of expected returns are negative and statistically significant (for instance, -0.39% for Pacific ex-Japan, -1.33% for Japan).[26] The bear regime tends to last from a minimum of 3.43 months in the BMINT data set to a maximum of 5.60 month in the IND data set (see Panels C and B of Table 4). The bull state generally gives high and often statistically significant expected returns, in 7 out of 8 portfolios in panel A, and for all the portfolios under examination in panels B and C. The persistence of the bull market regime is always higher than that of the bear state, which is fairly consistent with the evidence of asymmetric business cycles. The persistence of the bull regime is the highest in the IND data set (19.5 months) and the lowest in the INT data set (9.7 months). However, it is easy to realize that even persistence measures (average durations) between 3 and 4 months tend to be key to optimizing portfolio decisions, especially for investors with horizons relatively close to the implied average durations from the model estimates (say, up to 12-24 months). The conditional correlation/volatilities matrices are estimated with high precision under the two-state model, with a level of significance often inferior to 1%. Volatilities in the bear state are always larger than in the bull state, with the sole exception of EM Latin America in the INT data set.[27] These regularities sharply differentiate the two regimes in all the data sets examined in Table 4.

In panel A of Table 4, the two-regime representation leads to even greater dispersion across developed and EM equity portfolios than that already present in the single state representation. In the Bear regime, four out of eight markets have negative Sharpe ratios (Japan, Pacific ex-Japan, Europe ex-UK, UK), whereas Emerging Markets still have positive Sharpe ratios with EM Latin America displaying a particularly high Sharpe ratio of 0.428. North American stocks turn out to have the second highest Sharpe ratio, 0.090, with a relatively low volatility of 4.28%. In the Bull regime, volatilities are lower for every stock market but for EM Latin America, which is more volatile during Bull than Bear states. Nonetheless, the Sharpe ratio of EM Latin America (and also EM Europe & Middle East) exceeds 0.38, far higher than others—all of which are positive. Correlations involving North American and Europe ex-UK portfolios tend to be higher in bear markets, confirming the insight by Krishnan, Petkova, and Ritchken (2009) and Longin and Solnik (2001) that diversification may be more difficult in bear states. Panel C shows that in unconditional terms, the Bull regime is almost twice as likely as the Bear one (35% vs. 65%), an implication of the considerably higher bull persistence.

For the industry portfolios in panel B of Table 4, the Bull regime is more than three times as likely as the Bear one (78% vs. 22%).[28] In line with the positive mean returns of North American stocks in the

mimicking portfolios out of firm level data. By comparison, they reject both linearity and normality in both country and industry returns. A two-regime specification is the most suitable according to three information criteria (BIC, AIC, HQ).

[26]However, some exceptions can be found in panel C, where the bear regime is defined with reference to the properties of World market returns, while for a few portfolios, expected returns are higher in this state than in the (World) bull regime.

[27]This evidence is consistent with previous findings. For instance, Schwert (1989) and Hamilton and Lin (1996) indicate that the volatility of stock returns is higher during recessions than during expansions.This implies that in MS models it is common to find that high expected stock returns are associated wth low volatilities.

[28]This is broadly consistent with Catao and Timmermann (2007), where returns stay for 40 months in bear states and 42

IE data set, every mean US Industry return remains positive in the Bear regime, with a wide variation between Energy and Durables on the one end, respectively with mean monthly returns of 0.76% and 0.75%, and Other and Shops at the other end of the spectrum, with mean returns of 0.11% and 0.08%, respectively. Looking at volatilities, we note that Energy has a relatively low monthly volatility (9.3%) compared with that of, e.g., Durables (13.2%). Note that both correlations and volatilities are higher for industry than for country portfolios in the bear regime, suggesting that country diversification may be more powerful than industry diversification, as found by Griffin and Karolyi (1998). Under the Bull regime, the ranking of industry portfolios changes. Considering Sharpe ratios, the highest ratios obtain for Nondurables, Telecommunications and Health, all exceeding 0.28. All volatilities are lower under the Bull than under the Bear regime, while correlations are comparable across stock portfolios, with correlations being higher in the Bull than in the Bear state—even for "defensive" industries like Utilities.

In the case of panel C in Table 4 applied to BMINT data, EU ex-UK ex-Scandinavia Value has the highest Sharpe ratio (0.448). Bull-market correlations across Value portfolios range from 0.319 (United States/Scandinavia) to 0.562 (EU ex-UK ex-Scandinavia/ UK). Correlations across Growth portfolios are generally higher, ranging from 0.327 (United States/Asia & Pacific) to 0.626 (EU ex-UK ex-Scandinavia/ United States). Conditional correlations are similar to those of the single state model. In the Bear regime, UK Value and Asia & Pacific Value turn out to have the highest Sharpe ratios, respectively 0.246 and 0.222; whereas EU ex-UK ex-Scandinavia and Asia & Pacific growth the lowest, -0.035 and -0.018. Ang and Chen (2002) have studied the changing correlations across bull and bear states in US BM portfolios. We do confirm their result that US Value has higher correlation with the world portfolio than US Growth in bear markets. However, the same finding does not carry over to other international BM portfolios: value and growth stocks display a similar pattern of correlations across states, with all correlations being larger in bear states. Petkova and Zhang (2005) argue that the beta risk of value–minus–growth is higher in bear states: given that value stocks have far higher volatilities than growth stocks in bear markets, their finding carries over to our international data set. This also rationalizes why the expected return differential between value and growth stocks appears to be largest in Bear markets.[29]

Figure 1 provides information on the (smoothed, full-sample) probabilities of bull and bear regimes over our estimation periods. Also in this case, panels A-C correspond to smoothed probability estimates obtained from MSIH(2,0) models for the INT, IND, and BMINT data sets, respectively. In panel A, it is clear that international equity markets are most of the time in the Bull regime, although regime shifts occur relatively frequently, as implied by the Markov chain transition matrix estimates in panel A of Table 4. Bear market spells tend to last between 3 and 5 months, although some extended periods (e.g., mid-1997 to mid-1999) can be visualized. The exception is the sub-sample that goes from mid-2002 to mid-2007 which is characterized by a long Bull market that ends with the financial crisis of late 2007 and early 2008. Panel B, that refers to the IND data set, preserves the same qualitative features of regime shifts shown in panel A, although—because the IND sample is considerably longer (1926-2008)—the (incorrect) impression may be of more frequent switching. On the opposite, one can visualize very long Bear market spells: 1931-1934 (although more generally all of the 1930s are dominated by the bear state, which occupies approximately

in the normal one for country indexes vs 8 and 26 for industry indexes.

[29] For instance, the difference in mean returns between Asia & Pacific Value (1.17%) and Asia & Pacific Growth (0.61%) is 0.56% in bull markets, increasing to 1.74% in Bear markets when mean returns respectively equal 1.62% and -0.12%.

2/3 of the months in this decade) and 1998-2002. Also for IND, 2003-2007 is characterized by an extended Bull market period. Finally, panel C is interesting because it allows us to highlight the presence of bull and bear dynamics during the 1970s and the early 1980s. In particular, after a few spikes in 1976-1977, most of the long sub-sample 1978-1982 is characterized by a Bear world market state.

3.4. *Implied Moments from Bull and Bear Dynamic Models*

Table 5 performs a crucial operation: using Monte Carlo methods, we have computed the unconditional, long-run (co-)moments implied by our estimates of the two-state MSIH(2,0) models in panels A-C of Table 4. These implied moments are unconditional because obtained from a long, 100,000-period long simulation from the model to average out the dynamic properties of the data across bull and bear states in which the initial state clearly fails to carry any weight on the final estimate of the moments. These calculations are important because (8) is estimated by maximum-likelihood methods and not by matching the moments, while our economic application to portfolio choice revolves around the impact of moments and co-moments of equity returns on the (non-central) moments of the wealth process. Also Table 5 features three panels, A-C for INT, IND, BMINT, respectively, that need to be compared with panels A-C of Table 2. Although we refrain from computing summary measures of moment "fit" because this would make hardly any statistical sense in the presence of co-moments (such as co-skewness and co-kurtosis), the ability of our simple two-regime models to fit the sample moments of the date is excellent and offers further reassurance on the absence of extreme misspecifications of the process of returns.

In panel A, the only structural deviations concern a tendency of the bull and bear model to produce unconditional volatilities and kurtosis coefficients that are somewhat lower than in Table 2, although the differences are modest; moreover, also the co-skewness coefficients are often slightly above what we have found in Table 2 and this may imply that while the INT data set had very few significant co-skewness coefficients, the MS model tends to structurally magnify the co-skewness vs. the data, although differences remain modest. Panel B confirms these results on IND data: the bull and bear framework yields insufficient unconditional volatility, kurtosis, and especially co-kurtosis, even though the MS model outperforms a Gaussian IID framework that would trivially match unconditional volatility but miss completely the presence of excess kurtosis and co-kurtosis. The two-state model performs much better in terms of matching the skewness and co-skewness coefficients of the data, although some modest upward bias is visible. Finally, probably the best performance of a MS model at matching the unconditional moments of the data is probably revealed in panel C, with reference to the BMINT data set. Although, there is still some tendency to underestimate unconditional variance and kurtosis, in this case skewness and co-skewness coefficients are almost exactly matched.[30]

4. **Optimal Equity Portfolios**

We report qualitative portfolio results in Table 6. Before proceeding to comment the results in the table it is important to clarify its structure. Similarly to the entire structure of the paper, the table is organized around three panels, each for one of our three equity data sets. Within each panel, one can distinguish the

[30]One interesting implication of Table 5 is that the MSIH(2,0) model always fits mean stock returns almost exactly and that the model performs exceptionally well at matching ratios of moments, in this case correlations and Sharpe ratios.

results in the left- vs. the right-hand side of the table: on the left-hand side, we have results when no short sale constraints are imposed; on the right-hand side, no constraints have been imposed. Here, portfolio weights for 6 alternative horizons are reported, $T = 1, 3, 12, 24, 60$, and 120 months, as previously stated. The column "slope" also reports the difference between $T = 120$ and $T = 1$ month horizons, to given an overall and immediate idea for the type of effects on portfolio weights of a growing investment horizon.[31] In the left flank of the table, we have different alternative preference models: MV, MVS, MVSK, and MVK. Inside the table, different portions are devoted to alternative econometric frameworks, i.e., the single-state model with no predictability to be opposed to the two-regime bull and bear framework.[32]

In the case of the bull and bear model, optimal weights are computed under three alternative assumptions on the nature of the regime as of the date in which the optimization is solved: when the state is unknown and the investor assumes that the probability of the current state being Bear equals the unconditional, ergodic probability of the Bear regime; when the investor perceives the current regime to be Bear; when the investor perceives the current regime to be Bull. Importantly, these three alternative configurations not only span the range of possibilities for the perceived initial regime, they also have precise economic and econometric underpinnings. On the one hand, the case of an unknown initial regime stresses that in our paper the Markov state is unobservable and significant learning effects may occur over time. On the other hand, despite the latent nature of the Markov state, applied econometricians usually judge MS models also for their ability to produce "stark" times series of state (smoothed or filtered) probabilities that mostly oscillate between 0 and 1, i.e., to provide an accurate classification of the regimes from which the data may come at each point in time. In this sense, to initialize the bear state probability to either 0 or 1 appears to fit the common perception of a well-fitted MS model. In the following, we shall mostly comment on the classic MV(1,5) and MV(1,5)-c cases in which there is no predictability, on the cases where preferences are MV but there is bull and bear predictability (MV(2,5) and MV(2,5)-c), and on the case in which the investor cares for all moments of terminal wealth, MVSK(2,5) and MVSK(2,5)-c.

Appendix B describes the asset allocation results in detail. Here, to save space, we limit ourselves to a few general considerations. In the case of IE data, a classic mean-variance MV(1,5)-c investor only selects EM stocks, EM Latin America, and EM Europe & Middle East. This is not surprising because these two EM portfolios provide the highest Sharpe ratios across every investment horizon and exhibit a relatively low cross-correlation (0.48). When regimes are allowed, in the case of an unknown initial state, a MV(2,5)-c investor weighs positively North American stocks, especially when the horizon is short, along with EM Latin America and EM Europe & Middle East. Qualitatively, these implications hold also when the regimes are known: when the investor perceives an initial bear state, the portfolio is heavily tilted towards North American stocks and this is especially the case for short-horizon investor. In line with findings of Guidolin and Timmermann (2008a), this is explained by the relatively high (i.e., less negative) Sharpe ratio of US stocks in worldwide Bear states. In a bull regime, the optimal portfolio is

[31] Notice that because we do not allow for continuous, monthly rebalancing, this slope indicator cannot be interpreted as a hedging demand component in a technical sense. However, Guidolin and Timmermann (2008a) show how their approximate solution methods for portfolio optimization under MS may be extended to solve problems with frequent rebalancing over the investment period.

[32] In the single-state case, only results for MV preferences are displayed as a single-regime Gaussian IID implies constant (zero) co-skewness and excess co-kurtosis which are therefore absorbed in the constant of any Taylor expansion of a given utility function.

more diversified and qualitatively similar to the portfolio in which the initial regime is unknown. When the investor cares for all moments of terminal wealth, we observe a further increase in the degree of overall portfolio diversification vs. the MV(1,5)-c benchmark. However, North American, EM Latin American, and EM Europe & Middle East carry the largest weights, especially for long-run investors. The co-kurtosis between stock portfolios helps us rationalize these weights. Not only do MVSK(2,5)-c preferences lead to the highest level of diversification, but also to the most stable portfolio holding dynamics. These findings extend to the case in which short sales are unconstrained, even though in this case the spread among weights is obviously accentuated in the sense that the zero portfolio shares obtained above often turn into negative weights on the right-hand side of panel A of Table 6.[33]

Turning to the IND data set, the optimal portfolio of a MV(1,5)-c investor, is similar to that of an investor who also accounts for regimes, MV(2,5)-c. In fact, both strategies imply large portfolio shares in Energy, Health and Nondurables stocks, which have the highest Sharpe ratios. Considering the Bear and Bull states in isolation, Energy is overweighted in Bear states, as it has the highest mean return and a low volatility, whereas positions in Health and Nondurables increase in Bull markets, when they have the highest Sharpe ratios. When we allow for preferences that take into account the third and fourth moments of terminal wealth (MVSK(2,5)-c), the level of portfolio diversification slightly increases. Differently from MV(2,5)-c investors, MVSK(2,5)-c optimizers almost select the same allocation between the two regimes, with only Energy and Telecommunications receiving slightly higher weights in Bear states. Increasing the investment horizon to $T = 120$ alters the portfolio composition of the optimal MV(1,5)-c strategy, tilting it towards Telecommunications and away from Health. Also MV(2,5)-c investors tilt their portfolio allocation away from Health, but towards Nondurables. The MVSK(2,5)-c allocation lies in the middle of these two patterns, as it overweights Nondurables and under-weights both Health and Telecommunications.

Turning now to portfolio results for our third data set in panel C of Table 6, portfolio weights for MV(1,5)-c as well as MV(2,5)-c investors—when the regime is unknown—are concentrated in Value stocks for $T = 1, 12$, namely UK Value and Scandinavia Value, irrespective of the regimes. However, substantial differences appear in MV(2,5)-c strategies across bull and bear market states. Portfolios associated with four-moment preferences (MVSK(2,5)-c) display once more the highest degree of diversification. Accounting also for skewness and kurtosis induces the investor to diversify in 6 out of 11 portfolios, with Value stocks being more heavily weighted. When the horizon increases to $T = 60$, US Value and US Growth increase in importance to completely replace value portfolios for $T = 120$—when long-horizon returns are approximately Gaussian as a result of the central limit theorem.

4.1. *The Role of Co-Skewness and Co-Kurtosis*

It is interesting to investigate whether higher order moments explain why some stocks enter prominently in MV portfolios but have marginal, if any, role in MVS and MVSK portfolios. For instance, consider whether high order co-moments can explain the large reduction in weight attributed to EM Latin America (EMLA) and EM European & Middle East stocks in IE data when one switches from MV to MVS and MVSK strategies. Panel A of Table 5 shows that the EM Latin America portfolio has large negative values

[33] Li, Sarkar, and Wang (2003) have stressed how in international diversification issues, the case of no short sales should be considered as the leading one, given the obvious limits affecting the possibility to short emerging market stocks.

of both own-skewness ($S_{EMLA,EMLA,EMLA}$), and co-skewness coefficients, $S_{EMLA,EMLA,j}$ and $S_{EMLA,j,j}$ j = 1,..., 8, producing either the largest negative or second largest negative sample estimates of these moments. Hence EM Latin America stock returns tend to be negative when volatility is high in other markets and they are more volatile when other markets experience negative returns. The implication is that they provide little or no hedge against adverse returns or volatility shocks in other markets. A similar limitation affects the desirability of EM EU & Middle East stocks. These effects allow us to explain why aversion to skewness in the distribution of final wealth reduces the weights of these stock portfolios.

In the IND data set, we have noticed that the optimal portfolio becomes concentrated in Energy when we move from MV(2,5)-c to MVS(2,5)-c preferences, under a 1-month horizon. This can be traced back to the positive skewness of Energy returns (0.034) as well as to the negative skewness of both Nondurables (-0.147) and Health (-0.037). Co-skewness between portfolios are -0.092 and -0.067 (Energy/Nondurables), -0.010 and -0.012 (Energy/ Health), and -0.101 and -0.066 (Health/ Nondurables). Thus, there is a similarity between EM Latin America and EM Europe & Middle East and Health/ Nondurables relative to the role played by the North American and Energy stock portfolios. When considering MVSK(2,5)-c, the importance of both own-kurtosis and co-kurtosis help increase the number of stocks in the optimal portfolio selection. Indeed, Telecommunications and Energy, which are the portfolios in the highest demand, have the lowest own-kurtosis—respectively 5.761 and 5.315—and the lowest co-kurtosis, 2.744 with each other.

In the BMINT data set, Table 6 shows that investors with MVS(2,5)-c preferences avoid investments in Scandinavian value stocks that enter instead the MV(1,5)-c portfolios, along with UK value stocks. This can be traced back to the high own-skewness of Scandinavian value stocks which exceeds the one of UK Value (0.065 versus 0.0385), as well as to their negative co-skewness coefficients (-0.068 and -0.049). Table 5 also stresses that the implied unconditional co-kurtosis coefficients are the highest for UK Value/UK Growth pair (6.853). UK stocks are also characterized by high estimates of own-market kurtosis (6.92 and 10.34, respectively). The high value of own-kurtosis for UK Value stocks may explain why the allocation to this portfolio does not increase further when shifting from MVS ($m = 3$) to MVSK ($m = 4$) preferences.

More generally, it is easy to verify that the equity portfolios entering the optimal selection of MVSK investors have good co-kurtosis properties, with co-kurtosis coefficients ranging from 1.864 to 2.357. For instance, Asia Pacific Value stocks—that are not demanded at all under MV preferences—enter the optimal portfolios of both MVK and MVSK investors because they display a co-kurtosis of 1.902 with Scandinavian Value and of 2.357 with UK Value stocks. By comparison, US Growth, which dominates Asia Pacific Value in terms of implied Sharpe ratio (0.267 versus 0.184), has higher co-kurtosis (1.936 with Scandinavian Value stocks and 3.677 with UK Value stocks).

4.2. *Regularities Across Asset Menus*

Finally, it is useful to isolate any common features of the portfolio choices across the three data sets examined in the paper. A first regularity is that a MVSK(2,5)-c investor systematically achieves the highest degree of diversification. A MV(1,5)-c investor selects a highly concentrated allocation in which diversification opportunities seem to be left unexploited, holding for instance only two portfolios in the IE and BMINT data sets and three portfolios in the case of IND, at least as long as $T \leq 24$. A MV(2,5)-c investor selects instead six, three and two portfolios in the IE, IND and BMINT applications, and all these values are identical or exceed what we have found in the case of MV(1,5)-c. Even more, for a MVKS(2,5)-c, five, six, and eight (all of the available) portfolios are demanded independently of the investment horizon, in the case of IND, BMINT, and INT, respectively. This means that when preferences take into account the skewness and kurtosis of terminal wealth, the degree of diversification substantially increases.

Of course, this first common feature makes sense only in the case of constrained portfolios in which short sales are forbidden. A second pervasive feature of our results concerns instead all kinds of strategies: portfolio holdings in MV(1,5) models are heavily concentrated on stocks that yield the highest Sharpe ratios with lower correlations (especially among them), while stocks that yield low Sharpe ratios are sold short. Higher moment preferences result in the addition of those equity baskets with the lowest implied co-kurtosis and the highest co-skewness. However, increased diversification seem to follow especially from an aversion to terminal wealth kurtosis. This is clear from the comparison of MVS(2,5) and MVK(2,5) portfolios in the case of the IE data set: a higher number of international portfolios enters the optimal investor's selection when the investor has MVK(2,5) instead of MVS(2,5) preferences for every horizon but $T = 120$. Thus, it appears that skewness aversion leads to concentration in a subset of assets with good skewness properties—as already known from the literature.[34]

It is worthwhile stressing that MVSK(2,5) portfolios, despite being more diversified than MV(1,5) and MV(2,5), do not resemble equally weighted portfolios. First of all, the former usually displays zero investment in some of the assets, while $1/n$ clearly does not, by construction. When MVSK(2,5)-c portfolios do require long weights in all assets, as in the IE data set for $T = 1$, optimal portfolio shares range from 0.01 to 0.24, which are weights clearly deviating from the 1/8 rule. Furthermore, a MVSK(2,5) portfolio remains sensitive to the investment horizon, showing reduced diversification as T increases. In the IE data set, we find five non-zero holdings at $T = 60$ as opposed to 8 for $T = 1$; in the IND data set, we obtain four non-zero weights instead of 5; in the BMINT portfolio, we uncover four non-zero shares instead of six.

A third regularity concerns the volatility of portfolio holdings over time. Indeed, MV(2,5) strategies always entail more volatile weights, with frequent spikes, whose size increases with the investment horizon. On the other hand, under MVSK(2,5) strategies, these spikes in portfolio shares vanish in the IE data set and are anyway modest in both the IND and the BMINT asset menus. Therefore, transactions costs—which we have anyway left un-modelled up to this point—are more likely to adversely affect a MV rather than a four-moment investor and, when possible (i.e., should MV strategies ever out-perform MVSK ones in OOS tests), they could possibly reverse the ranking of models in term of realized ex-post performance. Ang and Bekaert (2004) have suggested that regime switching portfolio strategies are relatively robust to transaction costs because they are designed to exploit changes in expected returns and volatilities that are associated with infrequent changes of regimes. Our findings qualify this observation: a distinct contribution to the stability of portfolio shares is offered by the higher moments, given that the optimal weights exhibit a higher volatility under MV(2,5) than under MV(1,5).

As it is natural to expect (see, e.g., similar evidence in Guidolin and Timmermann, 2008b), it is also the case that a shorter horizon increases the sensitivity of optimal portfolio composition with respect to the current state of the market. For instance, a $T = 1$ investor in the IND application weights heavily equity portfolios that perform well in the Bear state when the probability of being in a Bear regime is high,

[34] With IE data, MVS(2,5) investors choose only the three highest Sharpe ratio portfolios, giving more weight to EM Europe & Middle East, which has instead the lowest skewness. Moreover, co-skewness among these portfolios is moderate.

because the Bear state always displays some appreciable degree of persistence. A MV(2,5)-c investor thus assigns a 0.612 weight to Energy stocks for $T = 1$. On the contrary, a $T = 120$ investor, believing to be in a Bear state, cares also about stock portfolios that outperform in Bull markets as she knows that the chances of shifting to Bull regimes are never negligible, given the properties of the Markov chain displayed in Table 4, panel B. Indeed, assuming $T = 120$, Non-Durable stocks receive a weight of 0.516, thanks to their attractiveness in bull states and despite their low mean return (0.162) in bear markets, while the weight on Energy stocks falls to 0.253.

5. **Ex-Post Performance Results**

We know that the ex-ante expected utility of an investor who cares about higher moments falls when she overlooks predictability in returns and/or higher order moments of the return distribution. This is, for instance, the case in Ang and Bekaert's (2002) and Guidolin and Timmermann's (2008a) international portfolio diversification problems, provided that the asset menu includes a short-term bond allowing investors to abandon equities in the bear state. It is also the case when dealing with size-sorted international equity portfolios, as in Guidolin and Nicodano (2009), due to the dismal performance of small caps in bear states. Moreover, ex-post realized OOS analyses confirm that gains are large when diversifying internationally if it is possibile to shift into cash in bear states (see e.g., Ang and Bekaert, 2004, and Guidolin and Timmermann, 2008a). In these papers the benchmark is the MV allocation with no predictability. However, we also know that ex-post gains from timing both volatility (e.g., Fleming *et al.*, 2001) and higher order moments (Jondeau and Rockinger, 2012) can be large, even when expected returns are held constant and there are no regimes. Therefore we now turn to an assessment of the OOS realized performance gains in our three data sets, extending previous evidence along several dimensions.

5.1. *Recursive Asset Allocation*

We recursively estimate by maximum likelihood all the parameters of the models described in Table 1 and proceed to calculate optimal portfolio shares and realized, recursive portfolio performance measures at all points in time over a selected (pseudo) OOS window. In practice, for the MS bull and bear framework, this is equivalent to estimating models obtaining outputs similar to those in Table 4, but on samples that are initially shorter than those used in Table 4 and that gradually expand to use all the available information. Importantly, given a sample $[t_0, t]$, optimal portfolio decisions with horizon T are always computed using estimated regime probabilities obtained as of time t, in a way consistent with the spirit of the exercise.

We use the following OOS expanding windows: 1998:01-2008:07 for the IE asset menu, 1980:01-2008:07 for IND, and 1995:01-2007:12 for BMINT. For instance, in the case of IND data, this means that we start by computing MS estimates and state probabilities using a 1926:07-1979:12 sample, using estimates and state probabilities as of the end of 1979 to compute portfolio shares with horizons $T = 1, 3, 6, 12, 24, 60$, and 120; the sample is then extended to 1926:07-1980:01 and estimations and asset allocation calculations are performed afresh; this process is iterated 343 times, until a final sample 1926:07-2008:07 is assembled.

Unreported plots of the dynamics of weights over time emphasizes that MV(1,5)-c offers the most stable allocation shares over time, whereas both MV(2,5)-c and MVSK(2,5)-c display large but infrequent spikes. Although the mapping is no way easy to visualize, there is a general tendency of shifts in portfolio weights to track the presence of regime switches in smoothed state probabilities in Figure 1. However, the inclusion of higher moments reduces the size of the spikes, which is lower for MVSK(2,5)-c.

5.2. *Realized Performance Measures*

We use five different indicators of portfolio performance: the classical Sharpe ratio, the Sortino ratio, the certainty equivalent return, the Treynor ratio, and the Jensen's alpha. The Sharpe ratio ($SR_{m,T} \equiv (\bar{r}_{m,T} - r^f T)/\hat{\sigma}_{m,T}$, for strategy m = MV, MVS, MVSK, MVK, etc., at horizon T) is widely used in the portfolio management literature and has the advantage of being independent of asset pricing models, relative to other common measures such as Jensen's alpha. However the Sharpe ratio is consistent with preferences only in the case of MV investors, because it disregards the high-order moments of terminal wealth. The Sortino ratio, defined as $SORT_{m,T} \equiv (\bar{r}_{m,T} - r^f T)/\hat{\sigma}^{down}_{m,T}$ (for strategy m = MV, MVS, MVSK, MVK, etc.) where $\hat{\sigma}^{down}_{m,T}$ is downside sample standard deviation defined as

$$\hat{\sigma}^{down}_{m,T} \equiv \left[\sum_{t=1}^{H} I_{\{r_{m\tau,T} < \bar{r}_{i,T}\}} \right]^{-1} \sum_{t=1}^{H} I_{\{r_{mt,T} < \bar{r}_{i,T}\}} (r_{mt,T} - \bar{r}_{m,T})^2, \tag{20}$$

where H is the number of periods for which we have computed realized portfolio measures.

However, both the Sharpe and the Sortino ratios provide no information on the skewness or kurtosis of wealth, being based on mean returns and (semi) sample standard deviations. They are therefore inadequate performance measures from the point of view of investors who care about higher-order moments. We may actually expect the Sharpe ratio achieved by MVS and MVSK strategies to be lower than the Sharpe ratio of MV strategies, because the Sharpe ratio may increase when portfolio returns display a more negative skew.[35] We therefore centre our comparisons on the (annualized) certainty equivalent of maximum utility, CEQ, associated with different investor preferences:

$$[1 + (T/12)(CEQ_{\mathcal{M},T}/100)]^{1-\gamma} = \left[\sum_{t=1}^{H-T} \left((\hat{\boldsymbol{\omega}}^{\mathcal{M}}_{t,T})' \exp(\mathbf{R}_{t,T}) \right)^{1-\gamma} \right], \tag{21}$$

where $\hat{\boldsymbol{\omega}}^{\mathcal{M}}_{t,T}$ is the vector of portfolio weights obtained from the combination of preferences/predictability model indexed by $\mathcal{M}$, $\mathbf{R}_{t,T}$ is the vector of asset returns over the interval $[t, t+T]$, and γ is the coefficient of constant relative risk aversion under which each of the preference structured has been obtained by a way of a Taylor expansion in (2). (21) may easily be solved to yield the annualized CEQ:

$$CEQ_{\mathcal{M},T} = \frac{1200}{T} \left[\sum_{t=1}^{H-T} \left((\hat{\boldsymbol{\omega}}^{\mathcal{M}}_{t,T})' \exp(\mathbf{R}_{t,T}) \right)^{1-\gamma} \right]^{\frac{1}{1-\gamma}} - 1.$$

Finally, the Treynor ratio and Jensen's alpha are well-known performance indices that assume the CAPM holds. Of course, when a (representative) investor has preferences defined over higher-order moments of terminal wealth, the CAPM will not hold (see Kraus and Litzenberger, 1977, and more recently Harvey and Siddique, 2000) and it is questionable whether it may be meaningful to report Treynor and alphas measures. Yet, given their prominence in the applied portfolio management literature, we also extend our evidence to these measures.[36]

[35] This is the idea behind he manipulation of Sharpe ratios discussed by Goetzmann et al. (2007).

[36] The calculations involving the Treynor ratio and the Jensen's alpha use the following portfolios as a proxy for the market

Table 7 (and C1 in Appendix C) is organized in a selective way: instead of reporting the realized performance for all the alternative preference and predictability models, we report performances for only four strategies: the $1/n$ benchmark always appears in the right-most column; the three best models in the set {MV(1,5), MV(2,5), MVS(2,5), MVK(2,5), MVSK(2,5)}. In this section, we over-weight strategies in which short-sale constraints are imposed. The reason is that, as seen in Table 6, when no constraints are imposed, our portfolios often imply a need of massive short positions that are neither easy to build in practice nor as cheap as we have assumed in this paper, where transaction costs have been ignored. We give priority to constrained strategies in the following way: the tables report on the best performing three strategies according each of the five performance criteria, and simply note when the ranking applies also to constrained strategies by adding a star ($*$). When the equally-weighted benchmark performs among the three best models according to a criterion, this is also noted in the right-most column of the tables. Finally, we compute 90% bootstrapped confidence bands for each of the performance measures.[37]

A first remark concerns the relative performance of the equally weighted benchmark vis-à-vis optimizing ones. Recently, the literature has suggested that the equally weighted strategy would be the appropriate benchmark for evaluating the relative performance of active strategies (DeMiguel et al., 2009). From this point of view, we observe that $1/n$ never consistently outperforms any of the optimizing strategies. According to the 90% confidence intervals, the equally weighted strategy is never the best performing one for all data data sets and is hardly ever among the three best performers in the case of IND data.

In the case of the INT asset menu (panel A), the MV(1,5) is the best performing strategy at all investment horizons (but one) under the Sharpe and Sortino criteria: in a pure MV perspective—even when the difference between total and downside variance is taken into consideration—it never pays out to track and forecast the bull and bear dynamics present in the data. However, when the criterion adopted is CEQ maximization, the picture is more complex and it seems that there is some advantage to be derived from predicting regimes and therefore from timing the dynamics in higher co-moments. Under a one-month horizon, the best performing strategy is MVSK(2,5) and this holds independently on the fact that short sale constraints are imposed. For $T = 12$, forecasting bull and bear states exploiting their persistence remains important, but a MV(2,5)-c preference framework seems to be sufficient to achieve a satisfactory performance. For $T = 60$, MVS(2,5) is the best performing strategy, but the distance from MV(2,5) is modest (12.3% per year vs. 12.2%). In this case, $1/n$ consistently performs well, but fails to be the best performing model, typically being ranked third at the shortest horizons.

These qualitative findings are confirmed by Appendix C that presents instead CAPM-related performance measures: MV(1,5)-c is hard to outperform in a Treynor ratio perspective, while the best performing model is more varied when one examines Jensen's alphas, especially at $T = 12$ and 60 months. All in all, although it is interesting to see that $1/n$ fails to outperform the active strategies across horizons and performance criteria, the punch-line is that for the purposes of equity portfolio diversification, the evidence in

portfolio: MSCI World market in the case of the INT and BMINT asset menus; value-weighted CRSP portfolio collecting data on NYSE, AMEX, and NADSAQ stocks for the IND data set. The Jensen's alpha is computed as the OLS intercept in a simple regression of excess realized portfolio returns on excess market returns.

[37] Following Guidolin and Timmermann (2006), we use a block bootstrap (with 50,000 independent trials) for the empirical distribution of each of the performance measures to account for the fact their realized levels are likely to be serially dependent as time-variations in the conditional distribution of asset returns may translate into dependencies in the portfolio weights and hence in realized performance measures.

favor of timing bull and bear markets and hence the dynamics in co-skewness and/or co-kurtosis is moderate: a simple, classical MV(1,5) framework performs well, especially at very short and very long investment horizons. Such good ex-post performance of MV(1) portfolios is perhaps not surprising, as we saw that non-normalities were moderate in INT data. Furthermore, we already know from Ang and Bekaert (2002) that the OOS cost of adopting Gaussian IID multivariate MV strategies in international diversification problems may be low when there is no risk-free rate explicitly included in the asset menu, as it is the case in our application. However, adopting a MV(2,5) strategy as opposed to a MV(1,5) may reward investors when performance assessment is performed using criteria that reflect the role of higher-order moments, in our case CEQ: for instance MV(2,5) delivers a CEQ of 4.99% rather than 4.71% from MV(1,5) assuming $T = 12$, and of 12.2% versus 9.6% when $T = 60$. Thus, accounting for regimes is rewarding for longer horizons even in this data set with moderate non-normalities. This complements prior results by Fleming et. al. (2001), who study a MV investor with a daily horizon. It also adds to the evidence in Jondeau and Rockinger (2012), who use a DCC specification for the return process.

The tone of the results in panel B of Table 7 is completely different and much easier to interpret. Here, the drastic divide is between short and long investment horizons. At $T = 1$ and 12 months, it always pays out to time bull and bear markets and—at least in some cases—specifically co-skewness and co-kurtosis. Although the best performing model is very often MV(2,5), at $T = 1$ the highest Sortino ratio is achieved by MVSK(2,5)-c (and by MVSK(2,5)) and at $T = 12$ the highest realized Sharpe ratio is achieved again by MVSK(2,5)-c (and by MVSK(2,5)). MVSK preferences also yield the best short-horizon performances according to the Treynor ratio and Jensen's alphas (see Appendix C). However, the picture turns mixed at a 60-month horizon—but in a CEQ perspective the highest realized utility is guaranteed by the MV(2,5) model—and favors simple, single-state MV models at a 10-year horizon. The importance of predicting both stock market regimes and higher order moments gets weaker in terms of Sharpe ratio with longer time horizons, but persists in terms of CEQ. The equally-weighted benchmark never represents a serious competitor: active portfolio strategies are always better than $1/n$ and in some cases such out-performance also has statistical back-up. For instance, at $T = 60$, the 90% confidence interval for the realized Sortino ratio of the best performing MVSK(2,5)-c model fails to overlap with the confidence interval for $1/n$. In the case of IND, despite substantial non-normalities both in our sample statistics in Table 2 as well as in the characterization of the return generating process in Table 4, the welfare maximizing strategy for all horizons is MV(2,5) which entails predicting regimes but ignoring higher order moments.

Panels C in Table 7 is important because—in a progression from the moderate importance of regimes and higher co-moments in INT, to the evidence of some importance of timing regimes (but not higher order moments) in the case of IND—marks a triumph of bull and bear strategies that actively time co-skewness and often also co-kurtosis when applied to the BMINT asset menu. Both MVSK(2,5)-c and MVSK(2,5) outperform any other strategy at a 1-month horizon, when we can deduce that both co-skewness and co-kurtosis should be timed for portfolio purposes. The differences between MVSK and the next best model (MV(2,5)-c) are large, for instance the Sharpe ratio is 1.32 vs. 1.19, the Sortino ratio is 1.86 vs. 1.59, and the annualized CEQ is 18.6% vs. 17.7%.[38] At long horizons, the differences between MVS and the next best model (often, MVSK(2,5)) are considerable and as a result the respective 90% bootstrapped confidence

[38] At $T = 12$, 60, and 120 months, it is instead MVS(2,5) that outperforms according to all the criteria, an indication that capturing bull and bear dynamics remains key, but only co-skewness should be actively timed.

bands fail to overlap. For instance, at $T = 60$, MVS(2,5)-c gives a Sharpe ratio of 0.57, a Sortino ratio of 1.25, and an annualized CEQ of 12.87% against realized values of 0.19, 0.81, and 3.60% for MVSK(2,5)-c. Such patterns and large differences vs. the second-best model are also shown in Table 8 with reference to the Treynor ratio and the Jensen's alpha. In the case of BMINT, the equally weighted benchmark turns out to be competitive, even though if we focus on investor's welfare, Table 7 reveals that the MVSK(2,5) strategy does better than $1/n$ in 8 cases out of 9, and in three cases significantly so. In other words, the MVSK strategy appears to be a resilient competitor of the equally weighted strategy across data sets. When diversifying within the BMINT asset menu, predicting stock market regimes continues to add value, as MV(2,5)-c systematically outperforms MV(1,5)-c. However, accounting for higher order preferences adds further economic value, as evident from panel C in Table 7. For instance, the welfare (CEQ) of an investor following the MV(2,5)-c strategy is lower than MVSK(2,5)-c: 17.70% vs. 18.64% at $T = 1$; 7.33% vs. 7.43% at $T = 12$; 3.38% vs.3.60% at $T = 60$.[39]

5.3. *Robustness Checks*

So far we have only commented results obtained assuming a constant coefficient of relative risk aversion of $\gamma = 5$ in the baseline power utility function used to derive moment-based preferences in (2). When commenting on Table 7 we have also focussed our attention on the case in which the asset allocation is constrained to avoid short positions. We first turn to the case in which $\gamma = 5$ but short sales are allowed. To save space, we did not tabulate results which are however available upon request. All three realized performance measures are generally lower than when short sales restrictions are imposed, particularly for long horizons. This is not surprising, as the typical extreme long or short positions involved by short selling are able to exacerbate any misspecification or errors caused by parameter estimation uncertainty, especially for long-run investment horizons. This result confirms previous research, which stressed the importance of restricting the volatility of portfolios weights to achieve higher realized OOS performance (e.g., DeMiguel et al., 2009, Diris et al., 2008, Jagannathan and Ma, 2003).

Allowing for short sales does not alter the relative ranking of performance when $T = 1$ in the INT menu, according to the performance measures reported in Table 7. Specifically, MV(1,5) is still the best model according to both Sharpe and Sortino ratios, while MVSK(2,5) yields the highest CEQ. Results are mixed when T increases to 12, with the ranking being the same only according to the Sharpe ratio that favours MV(1,5) also when short sales are allowed. On the contrary, IND data do not show the same patterns when we remove short sale constraints. When $T = 1$, the ranking changes both according to the Sharpe ratio and the CEQ, whereas MVSK(2,5) is consistently the best strategy in a Sortino ratio dimension. When $T = 12$, MVSK(2,5) and MV(1,5) are still the best and second best models according to the Sharpe ratio, while the ranking is completely changed for Sortino and CEQ measures. BMINT data show more stable rankings that are essentially unaffected by removing the short sale constraints.

The second robustness check consists in re-assessing realized OOS performances for $\gamma = 2$ and 10. To save space, we did not tabulate results which are however available upon request. Generally, the *relative* performance of the models is not strongly affected by the change in risk aversion coefficient. For instance, MV(1,2) and MV(1,10) remain the best strategies when applied on INT data and a 1-month horizon. Of

[39]Note that for $T = 60$ the bootstrapped 90% confidence bounds for annualized CEQ are not overlapping ($[11.28, 14.76]$ vs. [2.36, 4.58], respectively) and MVS(2,5) strictly dominates MV(2,5) also in a statistical sense.

course, what is changed is the absolute value of the various performance criteria, for instance the annualized CEQ that is higher (lower) for lower (higher) risk aversion coefficients, as the investor will dislike the even moments of terminal wealth more (less) and like the odd moments of wealth less (more). Relative results remain essentially unchanged in qualitative terms also in the case of IND and BMINT.

6. **Conclusions**

Using three relative large and realistic equity asset menus of both international and US domestic nature, in this paper we have found that—when the non-normalities induced by bull and bear dynamics as captured by a simple two-state MS model are sufficiently strong—portfolio strategies that exploit the presence of predictable (persistent) Bull and Bear market regimes and possibly time (forecast) co-skewness and co-kurtosis may steeply outperform simpler strategies that assume the absence of predictability and a multivariate Gaussian distribution for equity returns. In particular, we have uncovered large ex-post, realized OOS welfare (CEQ) gains from exploiting predictable moments up to the fourth order in international stock portfolios ranked according to BM ratios. This result is the mirror image, cast in a portfolio choice setting, of previous finance literature highlighting the bad performance of Value portfolios in bear states. Importantly, our simple MSIH(2,0) models deliver gains not only to an investor who cares about higher order moments, but also to an investor with MV preferences.

This result also holds when dealing with industry portfolios, at least for short horizons. Descriptive statistics point to large co-kurtosis across Industry portfolios. Despite this evidence, mean variance strategies that only exploit the persistence (hence, predictability) of bull and bear regimes perform better than those considering predictability in higher order moments. This fact begs for an explanation. Guidolin and Nicodano (2009) indicate that third (fourth) moments yield considerable (little) additional welfare in sample over a data set characterized by both types of non-normalities. We conjecture that MVSK($2,\gamma$) strategies do not outperform MV($2,\gamma$) strategies on industry data because these series display large co-kurtosis—i.e., high volatility when other stocks are also volatile—but little co-skewness. Further work should scrutinize whether the type of non-normality in the data affects ex-post gains from predicting higher order moments. Interestingly for both the BMINT and IND menus, we reported evidence that the equally weighted benchmark strategy has difficulties at outperforming active, optimizing strategies.

With reference to the international equity data set, our analysis also confirms previous results by Ang and Bekaert (2002): the OOS CEQ costs of adopting MV strategies may be low when an investor diversifies across international equities, with no opportunity to shift into bonds in bear states. We report, however, that even in this data set accounting for regimes and co-skewness (i.e., skewness in terminal wealth), especially for long horizons, can improve investors' realized welfare.

Our three recursive asset allocation experiments suggest that, at least to some extent (i.e., with adequate caution to be used in the case of INT data), modelling the regime switching nature of stock returns may be beneficial, but that higher order moments distinctly matter only in one out of three data sets. However, our empirical results can at most provide a lower bound to the relevance of higher order moments for portfolio strategies, as they are conditional on a specific parametrization of MVSK preferences. Their economic importance may be much larger for other types of preferences: for instance, allowing for investors' disappointment aversion, as in Hong et al. (2007), may boost gains from timing higher-order moments relative to the case of power utility.

In conclusion, allowing for a simple and parsimonious two-state MS representation of the return distribution (weakly) improves on ex-post investors' welfare for all horizons and in all data sets. Predicting third and fourth moments, on top of the first two, need not always deliver gains even when descriptive statistics indicate the presence of sizeable non-normalities. We are currently scrutinizing these results through formal statistical tests to try and anticipated in ex-ante terms when and how any empirical evidence of excess co-skewness or co-kurtosis vs. a standard Gaussian benchmark, may reveal the potential for ex-post OOS improved portfolio performance. Moreover, the inclusion of ex-post transaction costs, which we leave for future work, may further increase the relative attractiveness of predicting higher order moments on top of timing bull and bear markets, because when both co-skewness and co-kurtosis are timed, portfolio shares appear to be less sensitive to variations in expected returns.

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Table 1

List of Asset Allocation Models

This table lists the asset allocation models we compare in the paper. The last column of the table gives the acronyms used to refer to the strategies in the following tables, where we compare the performance of the asset allocation models. Besides the 31 alternative models appearing in the table, whenever possible for each of them we perform portfolio optimization for six different time horizons, listed in the fifth column.

	Preferences	Predictability Model	Short Sales Allowed	"Deep" Risk Aversion Coeff.	Horizons (in months)	Abbreviation
	Classical Mean-Variance Models with No Predictability					
1-3	Mean-Variance	No Predictability (Gaussian IID)	Yes	2, 5, 10	1, 3, 6, 12, 24, 60, 120	MV(1,2), MV(1,5), MV(1,10)
4-6	Mean-Variance	No Predictability (Gaussian IID)	No	2, 5, 10	1, 3, 6, 12, 24, 60, 120	MV(1,2)-c, MV(1,5)-c, MV(1,10)-c
	Mean-Variance Models with MS Predictability					
7-9	Mean-Variance	MSIH(2,0)	Yes	2, 5, 10	1, 3, 6, 12, 24, 60, 120	MV(2,2), MV(2,5), MV(2,10)
10-12	Mean-Variance	MSIH(2,0)	No	2, 5, 10	1, 3, 6, 12, 24, 60, 120	MV(2,2)-c, MV(2,5)-c, MV(2,10)-c
	Higher-Moment Preference Models with MS Predictability					
13-15	Mean-Variance-Skweness	MSIH(2,0)	Yes	2, 5, 10	1, 3, 6, 12, 24, 60, 120	MVS(2,2), MVS(2,5), MVS(2,10)
16-18	Mean-Variance-Skewness	MSIH(2,0)	No	2, 5, 10	1, 3, 6, 12, 24, 60, 120	MVS(2,2)-c, MVS(2,5)-c, MVS(2,10)-c
19-21	Mean-Variance-Kurtosis	MSIH(2,0)	Yes	2, 5, 10	1, 3, 6, 12, 24, 60, 120	MVK(2,2), MVK(2,5), MVK(2,10)
22-24	Mean-Variance-Kurtosis	MSIH(2,0)	No	2, 5, 10	1, 3, 6, 12, 24, 60, 120	MVK(2,2)-c, MVK(2,5)-c, MVK(2,10)-c
25-27	Mean-Variance-Skweness-Kurtosis	MSIH(2,0)	Yes	2, 5, 10	1, 3, 6, 12, 24, 60, 120	MVSK(2,2), MVSK(2,5), MVSK(2,10)
28-30	Mean-Variance-Skewness-Kurtosis	MSIH(2,0)	No	2, 5, 10	1, 3, 6, 12, 24, 60, 120	MVSK(2,2)-c, MVSK(2,5)-c, MVSK(2,10)-c
31	Equally Weighted Portfolio	_____	No	______	1, 3, 6, 12, 24, 60, 120	1/N

Table 2

Summary Statistics for Equity Returns

The table reports basic moments for monthly equity total return series for international portfolios from January 1988 to July 2008 (Panel A), Industries indices from July 1926 to July 2008 (Panel B) and International Book-to-Market portfolios from January 1975 to December 2007(Panel C) in the upper part of each panel. All returns are expressed in local currencies. Means, Median and Standard Deviations are annualized. The column Jarque-Bera reports the value of the Jarque-Bera statistics for normality, while LB(12) reports the 12^{th}-order Ljung-Box statistic. The middle part of each panel reports the correlation and co-kurtosis matrices, the lower part the co-skewness matrix. In the co-skewness matrix, coefficients above the main diagonal refer to the sample covariance between the square of the returns of the row portfolio and the level of returns of the column portfolio; coefficients below the main diagonal refer to the sample covariance between the level of the returns of the column portfolio/index and the square of returns of the row portfolio/index. In the correlation/co-kurtosis matrix, correlations are reported above the main diagonal and sample covariances between squared portfolio returns appear below the main diagonal. The symbols ** and * respectively denote statistical significance at 1% and 5%.

Panel A (International MSCI USD Returns, 1988:01 - 2008:07)

	Mean	St. Dev.	Sharpe ratio	Median	Min.	Max.	Skewness	Kurtosis	Jarque-Bera	LB(12)	LB(12)-squares
Pacific ex-Japan	0.746*	5.428**	0.072	0.927	-23.1	15.3	-0.530*	4.685*	40.94**	18.62	28.59**
Japan	-0.051	6.310**	-0.064	-0.272	-21.6	21.7	0.101	3.696	5.43	10.94	52.51*
Europe ex-UK	0.766*	4.928**	0.084	1.103	-15.6	13.8	-0.542*	4.059	23.73**	15.48	25.22*
United Kingdom	0.707	4.397**	0.080	0.673	-10.9	14.1	0.038	3.178	0.389	11.23	57.88**
North America	0.756**	3.924**	0.103	1.093	-14.3	10.4	-0.441*	3.714	13.309**	7.77	34.88**
EM Latin America	1.906**	8.933**	0.174	2.680	-35.4	27.3	-0.594*	4.536*	38.985**	10.97	20.72
EM Asia	0.774	7.111**	0.059	1.078	-19.7	22.1	-0.181	3.717	6.657*	33.06**	45.27**
EM Europe & Middle East	1.846**	7.747**	0.193	2.450	-29.0	38.8	0.272	5.888**	89.256**	14.75	5.40

Correlation and (variance) co-kurtosis matrices

	Pacific ex-JP	Japan	EU ex-UK	UK	North Amr.	EM Latin Amr.	EM Asia	EM EU & Middle East
Pacific ex-JP		0.444**	0.592**	0.621**	0.601**	0.545**	0.785**	0.424**
Japan	1.292		0.462**	0.480**	0.368**	0.321**	0.406**	0.221**
EU ex-UK	1.894	1.715		0.744**	0.669**	0.410**	0.508**	0.467**
UK	1.533	1.459	2.317*		0.664**	0.394**	0.442**	0.353*
North Amr.	1.766	1.346	2.801*	2.138**		0.500**	0.551**	0.404**
EM Latin Amr.	1.923	1.806	2.012*	1.379	2.255		0.491**	0.479**
EM Asia	3.149**	1.440	2.114	1.488	1.979	1.826		0.475**
EM EU & Middle East	1.407	1.503	2.000	1.384	2.250	2.286	1.586	

Co-skewness matrix

	Pacific ex-JP	Japan	EU ex-UK	UK	North Amr.	EM Latin Amr.	EM Asia	EM EU & Middle East
Pacific ex-JP		-0.273	-0.360	-0.203	-0.352*	-0.385	-0.364	-0.299*
Japan	-0.159		-0.052	0.071	-0.078	-0.307	-0.160	-0.115
EU ex-UK	-0.374**	-0.306		-0.351	-0.496*	-0.354	-0.470**	-0.344
UK	-0.043	-0.090	-0.178		-0.080	-0.117	-0.118	-0.187
North Amr.	-0.393**	-0.231	-0.520**	-0.238		-0.475*	-0.410*	-0.419
EM Latin Amr.	-0.400**	-0.420*	-0.213	-0.271	-0.380		-0.269	-0.337
EM Asia	-0.292	-0.263	-0.372*	-0.209	-0.273	-0.277		-0.288*
EM EU & Middle East	-0.220	-0.131	-0.126	-0.218	-0.344	-0.565	-0.260	

Table 2 (continued)

Summary Statistics for Equity Returns

Panel B (CRSP Industry Returns, 1926:07 - 2008:07)

	Mean	St. Dev.	Sharpe ratio	Median	Min.	Max.	Skewness	Kurtosis	Jarque-Bera	LB(12)	LB(12)-squares
Non Durables	0.978**	4.691**	0.143	1.090	-24.5	34.4	-0.029	8.843**	1401.5**	36.17**	333.9**
Durables	1.074**	7.593**	0.101	1.000	-34.8	79.7	1.203	18.42*	9993.4**	51.38**	266.4**
Manufacturing	1.034**	6.322**	0.115	1.330	-29.8	57.4	0.978	15.59**	6761.2**	39.407**	440.3**
Energy	1.097**	5.983**	0.132	0.860	-26.0	33.5	0.238	6.183**	425.02**	23.27*	251.4**
Hi Tech	1.094**	7.437**	0.106	1.220	-33.8	53.4	0.296	9.030**	1506.7**	26.91**	562.3**
Telecommunications	0.831**	4.594**	0.115	0.880	-21.6	28.2	0.056	6.277**	441.18**	29.49**	342.2**
Shops/Distribution	0.975**	5.884**	0.114	1.130	-30.2	37.1	-0.016	8.501**	1242.0**	56.72**	458.6**
Health	1.089**	5.766**	0.136	1.070	-34.7	38.7	0.171	10.210**	2136.2**	53.59**	605.9**
Utilities	0.902**	5.685**	0.105	1.050	-33.0	43.2	0.095	10.61**	2379.7**	52.12**	622.3**
Other	0.921**	6.473**	0.095	1.260	-30.0	58.7	0.971	16.83**	8006.3**	64.55**	490.0**

Correlation and (variance) co-kurtosis matrices

	Non Durables	Durables	Manufacture	Energy	Hi Tech	Telecom	Shops	Health	Utilities	Other
Non Durables		0.754**	0.851**	0.616**	0.736**	0.671**	0.866**	0.801**	0.707**	0.847**
Durables	10.490*		0.873**	0.607**	0.779**	0.618**	0.798**	0.649**	0.635**	0.802**
Manufacturing	10.150**	15.873*		0.723**	0.862**	0.671**	0.841**	0.762**	0.703**	0.905**
Energy	5.297**	7.095**	7.328**		0.607**	0.495**	0.576**	0.562**	0.617**	0.689**
Hi Tech	7.546**	11.109*	10.872**	5.273**		0.676**	0.785**	0.723**	0.624**	0.798**
Telecommunications	4.614**	5.257**	5.930**	3.265**	5.248**		0.670**	0.600**	0.635**	0.695**
Shops/Distribution	7.880**	9.697**	9.688**	4.810**	7.499**	4.878**		0.740**	0.655**	0.824**
Health	8.130**	9.952**	10.163**	4.987**	7.842**	5.079**	7.860**		0.625**	0.741**
Utilities	6.660**	8.297**	9.188**	4.686**	7.514**	5.839**	7.502**	7.707**		0.740**
Other	8.496**	10.936**	13.017**	6.500**	9.662**	6.876*	8.927**	9.863**	10.436**	

Co-Skewness matrices

	Non Durables	Durables	Manufacture	Energy	Hi Tech	Telecom	Shops	Health	Utilities	Other
Non Durables		0.248	0.196	-0.032	0.058	-0.153	-0.054	-0.021	-0.077	0.046
Durables	0.633		1.042	0.554	0.754	0.164	0.577	0.615	0.397	0.663
Manufacturing	0.505	0.981		0.459	0.684	0.259	0.477	0.600	0.472	0.777
Energy	-0.013	0.248	0.239		0.128	-0.052	-0.064	0.118	0.029	0.180
Hi Tech	0.172	0.451	0.460	0.156		-0.017	0.167	0.234	0.155	0.333
Telecommunications	-0.093	-0.085	0.043	-0.126	-0.030		-0.096	-0.041	0.064	0.109
Shops/Distribution	-0.051	0.228	0.187	-0.058	0.069	-0.104		-0.051	-0.024	0.119
Health	0.061	0.325	0.363	0.150	0.192	0.030	0.037		0.065	0.311
Utilities	-0.046	0.120	0.228	-0.032	0.098	0.089	-0.020	0.009		0.302
Other	0.337	0.658	0.802	0.380	0.555	0.370	-0.020	0.555	0.579	

Table 2 (continued)

Summary Statistics for Equity Returns

Panel C (International Book-to-Market Sorted Portfolio Local Returns, 1975:01 - 2007:12)

	Mean	St. Dev.	Sharpe ratio	Median	Min.	Max.	Skewness	Kurtosis	Jarque-Bera	LB(12)	LB(12)-squares
World	0.694**	3.844**	0.055	1.000	-22.0	12.8	-0.985**	6.832	306.4**	13.88	6.851
EU ex-UK ex-Scand Value	1.326**	4.740**	0.178	1.640	-18.7	16.4	-0.486*	5.130*	90.47**	24.06*	22.72*
EU ex-UK ex-Scand Growth	0.967**	4.294**	0.112	1.280	-24.9	14.9	-0.733	6.915	288.4**	18.78	15.89
United Kingdom Value	1.725**	6.187**	0.201	1.700	-27.0	45.5	0.845	10.95	1088.9**	13.80	44.02**
United Kingdom Growth	1.353**	5.851**	0.148	1.325	-27.9	53.8	1.610*	20.44*	5186.8**	12.60	11.83
Asia & Pacific Value	1.300**	5.123**	0.159	1.000	-25.0	19.1	-0.095	5.615*	113.4**	11.57	56.82**
Asia & Pacific Growth	0.401	4.936**	-0.017	0.525	-18.4	25.1	-0.003	5.214	80.88**	11.21	81.93**
Scandinavia Value	1.676**	6.428**	0.185	1.765	-22.1	25.8	0.175	4.358*	32.46**	29.48**	25.14*
Scandinavia Growth	1.486**	6.270**	0.160	1.770	-21.4	25.5	0.037	4.763**	51.36**	22.75*	80.34**
United States Value	1.081**	4.769**	0.125	1.255	-24.3	14.2	-0.473	4.901	74.41**	7.301	11.83
United States Growth	1.448**	4.302**	0.224	1.660	-20.4	23.7	-0.177	7.616**	353.7**	19.06	29.92**

Correlation and (variance) co-kurtosis matrices

	World	EU ex-UK ex-Scand Value	EU ex-UK ex-Scand Growth	UK Value	UK Growth	Asia Pacific Value	Asia Pacific Growth	Scandinavia Value	Scandinavia Growth	United States Value	United States Growth
World		0.742**	0.792**	0.632**	0.644**	0.593**	0.681**	0.525**	0.625**	0.859**	0.804**
EU ex-UK ex-Scand Value	4.624**		0.850**	0.573**	0.506**	0.470**	0.441**	0.584**	0.542**	0.532**	0.627**
EU ex-UK ex-Scand Growth	6.160**	4.682**		0.550**	0.584**	0.434**	0.513**	0.499**	0.619**	0.621**	0.593**
United Kingdom Value	4.901**	3.204**	4.632**		0.786**	0.403**	0.389**	0.419**	0.377**	0.464**	0.556**
United Kingdom Growth	5.620**	3.397**	5.560*	13.637*		0.317**	0.357**	0.370**	0.407**	0.546**	0.563**
Asia & Pacific Value	4.452**	3.157**	4.368*	3.107**	2.962*		0.649**	0.412**	0.332**	0.337**	0.360**
Asia & Pacific Growth	2.995**	2.047**	2.811**	1.837**	1.806**	3.071**		0.344**	0.454**	0.409**	0.350**
Scandinavia Value	2.827**	2.515**	2.820**	2.149**	2.162**	2.325**	1.620**		0.643**	0.372**	0.432**
Scandinavia Growth	3.330**	2.453**	3.569**	2.132**	2.200**	2.267**	1.989**	2.160**		0.546**	0.434**
United States Value	5.050**	3.166**	4.520*	3.428**	3.819*	3.078	1.873**	2.223**	2.789**		0.784**
United States Growth	5.582**	3.727**	4.887*	6.251*	8.375*	3.082*	1.748**	2.256**	2.219**	4.460**	

Co-Skewness matrices

	World	EU ex-UK ex-Scand Value	EU ex-UK ex-Scand Growth	UK Value	UK Growth	Asia Pacific Value	Asia Pacific Growth	Scandinavia Value	Scandinavia Growth	United States Value	United States Growth
World		-0.837*	-0.976*	-0.516	-0.499	-0.829*	-0.572*	-0.650*	-0.681*	-0.797	-0.733
EU ex-UK ex-Scand Value	-0.661*		-0.595*	-0.359	-0.416	-0.501*	-0.335*	-0.437*	-0.433*	-0.564*	-0.552*
EU ex-UK ex-Scand Growth	-0.881	-0.685		-0.536	-0.499	-0.771	-0.445	-0.642*	-0.552	-0.783	-0.741
United Kingdom Value	-0.040	-0.116	-0.119		0.994	-0.129	-0.049	-0.249	-0.208	-0.097	0.340
United Kingdom Growth	-0.138	-0.028	0.048	1.236		-0.199	-0.041	-0.168	-0.056	0.015	0.589
Asia & Pacific Value	-0.583	-0.410	-0.528	-0.278	-0.404		-0.239	-0.436	-0.437	-0.524	-0.533
Asia & Pacific Growth	-0.301	-0.205	-0.197	-0.097	-0.148	-0.154		-0.329**	-0.307*	-0.259*	-0.285*
Scandinavia Value	-0.339	-0.317	-0.366	-0.191	-0.280	-0.216	-0.123		-0.109	-0.309	-0.300
Scandinavia Growth	-0.389	-0.349*	-0.353	-0.308	-0.231	-0.399*	-0.243	-0.134		-0.225	-0.324
United States Value	-0.644	-0.635**	-0.790*	-0.376	-0.407	-0.583	-0.360*	-0.437	-0.424		-0.451
United States Growth	-0.501	-0.559	-0.668	0.017	0.006	-0.489	-0.343	-0.424	-0.412	-0.415	

** Statistical significance at 1%; * Statistical significance at 5%.

Table 3

Model Selection Statistics

The table reports estimates for the multivariate Markov switching conditionally heteroskedastic VAR model:

$$\mathrm{r}_t = \mu_{S_t} + \sum_{i=1}^{p} \mathrm{A}_{i,S_t} \mathrm{r}_{t-i} + \varepsilon_t$$

where μ_{S_t} is the intercept vector in state S_t, A_{i,S_t} is the matrix of autoregressive coefficients associated with lag $i \geq 1$ in state S_t and $\varepsilon_t = (\varepsilon_{1t},...\varepsilon_{ht})'$~ N($\mathbf{0}, \Omega_{S_t}$). The unobserved state variable S_t is governed by a first-order Markov chain that can assume *k* distinct values. *p* autoregressive terms are considered. The sample period is 1988:01-2008:08 for Panel A (International portfolios), 1926:07-2008:07 for Panel B (Industries) and 1975:01-2007:12 for Panel C (Book-to-Market). MISIAH(*k*,*p*) stands for Markov Switching Intercept Autoregressive Heteroskedasticity Model with *k* states and *p* autoregressive lags. The third and fourth columns report the likelihood ratio statistic and the corresponding adjusted p-value for the null of one regime against the alternative of *k* > 1 regimes, using Davies' approximation. The last column of the table performs likelihood ratio tests (for given number of regimes) of the following restrictions: H, no regimes in the covariance matrix; I, no regimes in the intercept vector; VAR, no regimes in the matrix of vector autoregressive coefficients. In the column, we report the LR test statistic and the corresponding p-values with a number of degrees of freedom equal to the number of restrictions imposed of the type H, I, and VAR.

Panel A (International MSCI USD Returns, 1988:01 - 2008:08)

Model (K,p)	Log-likelihood	LR Statistic	Davies' approx. p-value	BIC	HQ	AIC	Number of parameters	Number of obs.	Saturation ratio	Tests
International MSCI USD Returns, 1988:01 - 2008:07										
	Single-state models									
MSIA(1,0)	3350.92	__	__	**-25.978**	-26.450	-26.701	44	1984	45.1	
MSIA(1,1)	3530.31	__	__	-23.723	-25.573	-26.820	108	1976	18.3	
	Two-state models									
MSI(2,0)	3375.97	50.089	0.000	**-26.025**	-26.482	-26.790	54	1984	36.7	
MSIH(2,0)	3450.75	199.665	0.000	-25.828	**-26.790**	-27.103	90	1984	22.0	H: 149.58 (0.000)
MSH(2,0)	3334.33	138.826	0.000	**-26.075**	-26.769	**-27.237**	82	1984	24.2	I: 232.84 (0.000)
MSIA(2,1)	3501.38	219.044	0.000	-24.292	-25.837	-26.878	182	1976	10.9	VAR: 250.82 (0.000)
MSIAH(2,1)	3530.31	276.913	0.000	-23.723	-25.573	-26.820	218	1976	9.1	H: 57.87 (0.012)
	Three-state models									VAR: 159.12 (0.032)
MSI(3,0)	3385.22	68.591	0.000	-25.665	-26.424	-26.800	66	1984	30.1	
MSIH(3,0)	3570.60	439.350	0.000	-25.727	**-26.895**	**-27.682**	138	1984	14.4	
MSIA(3,1)	3576.25	368.785	0.000	-23.203	-25.393	-26.868	258	1976	7.7	
MSIAH(3,1)	3666.89	550.075	0.000	-22.331	-25.132	-27.019	330	1976	6.0	
	Four-state models									
MSI(4,0)	3415.33	128.813	0.000	-25.764	-26.442	-26.898	80	1984	24.8	
MSIH(4,0)	3647.38	592.912	0.000	-25.235	**-26.826**	**-27.898**	188	1984	10.6	
MSIA(4,1)	3669.26	554.801	0.000	-22.216	-25.068	-26.990	336	1976	5.9	

Table 3 (continued)

Model Selection Statistics

Panel B (CRSP Industry Returns, 1926:07 - 2008:07)

Model (K,p)	Log-likelihood	LR Statistic	Davies' approx. p-value	BIC	HQ	AIC	Number of parameters	Number of obs.	Saturation ratio	Tests
CRSP Industry Returns, 1926:07 - 2008:07										
	Single-state models									
MSIA(1,0)	19354.00	__	__	-38.843	-39.043	-39.166	65	9850	151.5	
MSIA(1,1)	19481.34	__	__	-38.441	-38.949	-39.261	165	9850	59.7	
	Two-state models									
MSI(2,0)	19428.38	148.767	0.000	-38.910	-39.147	-39.292	77	9850	127.9	H: 2028.35 (0.000)
MSIH(2,0)	20442.56	2177.118	0.000	**-40.391**	**-40.797**	-41.047	132	9850	74.6	
MSH(2,0)	20242.57	1777.147	0.000	**-40.344**	-40.720	-40.950	122	9850	80.7	I: 399.97 (0.000)
MSIA(2,1)	19943.31	923.937	0.000	-38.595	-39.448	-39.972	277	9840	35.5	
MSIAH(2,1)	20514.06	2065.444	0.000	-39.370	-40.393	-41.020	332	9840	29.6	
	Three-state models									
MSI(3,0)	19532.68	357.374	0.000	-39.024	-39.476	-39.304	91	9850	108.2	
MSIH(3,0)	20504.22	2300.450	0.000	**-40.226**	-40.845	**-41.225**	201	9850	49.0	
MSIA(3,1)	20106.58	1250.476	0.000	-38.129	-39.333	-40.072	391	9840	25.2	
MSIAH(3,1)	20704.65	2446.620	0.000	-38.574	-40.117	-41.064	501	9840	19.6	
	Four-state models									
MSI(4,0)	19607.79	507.578	0.000	-39.064	-39.393	-39.596	107	9850	92.1	
MSIH(4,0)	20423.45	2138.898	0.000	-40.136	**-40.973**	**-41.487**	272	9850	36.2	
MSIH(4,0)-VAR(1)	20826.51	2690.348	0.000	-39.725	**-40.871**	**-41.574**	372	9840	26.5	

Table 3 (continued)

Model Selection Statistics

Panel C (International Book-to-Market Sorted Portfolio Local Returns, 1975:01 - 2007:12)

Model (K,p)	Log-likelihood	LR Statistic	Davies' approx. p-value	BIC	HQ	AIC	Number of parameters	Number of obs.	Saturation ratio	Tests
International Book-to-Market Sorted Portfolio Local Returns, 1975:01 - 2007:12										
Single-state models										
MSIA(1,0)	8772.32	__	__	-43.142	-43.609	-43.916	77	4356	56.6	
MSIA(1,1)	8929.72	__	__	-41.171	-43.190	-44.516	198	4345	21.9	
Two-state models										
MSI(2,0)	8830.76	116.877	0.000	**-43.240**	-43.787	-44.145	90	4356	48.4	H: 505.68 (0.000)
MSIH(2,0)	9083.60	622.555	0.000	**-43.521**	**-44.468**	-45.089	156	4356	27.9	
MSH(2,0)	9038.61	532.590	0.000	**-43.513**	**-44.389**	-44.965	145	4356	30.0	I: 89.98 (0.000)
MSIA(2,1)	9123.80	388.169	0.000	-41.171	-43.190	-44.516	332	4345	13.1	
MSIAH(2,1)	9240.63	621.826	0.000	-40.764	-43.184	-44.773	398	4345	10.9	H: 233.66 (0.000)
Three-state models										
MSI(3,0)	8854.06	163.463	0.000	-43.132	-43.769	-44.187	105	4356	41.5	
MSIH(3,0)	9212.01	879.363	0.000	-42.946	-44.384	**-45.328**	237	4356	18.4	
MSIA(3,1)	9344.81	830.185	0.000	-40.232	-43.078	-44.946	468	4345	9.3	
MSIAH(3,1)		No converge achieved (too many parameters)					600	4345	7.2	
Four-state models										
MSI(4,0)	8894.16	243.666	0.000	-43.077	-43.818	-44.304	122	4356	35.7	
MSIH(4,0)	9375.96	1207.270	0.000	-42.520	**-44.463**	**-45.737**	320	4356	13.6	
MSIH(4,0)-VAR(1)	9452.93	1046.411	0.000	-41.188	-43.870	**-45.630**	441	4345	9.9	

Table 4

Estimated Markov Switching Models

The table shows estimation results for the regime switching model

$$r_t = \mu_{S_t} + \varepsilon_t.$$

r_t is the vector collecting monthly total return series, μ_{S_t} is the intercept vector in state S_t and $\varepsilon_t = (\varepsilon_{1t},...\varepsilon_{ht})'$~ $N(\mathbf{0}, \Omega_{S_t})$. Panel A shows the estimation results for the International portfolios dataset, Panel B for the Industries one, Panel C for the Book-to-Market portfolios one. Each Panel reports the results for the single-state model, k = 1, (Panel A), for the two-state model k = 2 (Panel B) and its Markov chain properties (Panel C) (ergodic probabilities and average state duration).

Panel A (International MSCI USD Returns, 1988:01 - 2008:07)

	SINGLE STATE MODEL							
	Pacific EX JP	Japan	Europe EX UK	UK	North America	EM Latin America	EM Asia	EM Europe & Middle East
1. Mean returns	0.746*	-0.051	0.766*	0.707*	0.756**	1.906**	0.774	1.846**
2. Correlations/Volatilities								
Pacific EX JP	5.428**							
JP	0.444**	6.310**						
Europe EX UK	0.592**	0.462**	4.928**					
UK	0.621**	0.480**	0.744**	4.397**				
North America	0.601**	0.368**	0.669**	0.664**	3.924**			
EM Latin America	0.545**	0.321**	0.410**	0.394**	0.500**	8.932**		
EM Asia	0.785**	0.406**	0.508**	0.442**	0.551**	0.491**	7.111**	
EM Europe and Middle East	0.424**	0.221**	0.467**	0.352**	0.404**	0.479**	0.475**	7.747**
	TWO-STATE MODEL							
	Pacific EX JP	JP	Europe EX UK	UK	North America	EM Latin America	EM Asia	EM Europe & Middle East
1. Mean returns								
Bear/High Volatility State	-0.386*	-1.325**	-0.087	-0.051	0.384	0.496	0.336	0.227
Bull/Low Volatility State	1.371**	0.653	1.238**	1.125**	0.961**	2.685**	1.017*	2.740**
2. Correlations/Volatilities								
Bear/High Volatility State								
Pacific EX JP	7.081**							
JP	0.399**	7.524**						
Europe EX UK	0.498**	0.473**	5.850**					
UK	0.554**	0.514**	0.772**	4.478**				
North America	0.561**	0.355**	0.621**	0.509**	4.284**			
EM Latin America	0.514**	0.336*	0.234*	0.365*	0.475**	1.159**		
EM Asia	0.779**	0.365**	0.435**	0.364**	0.550**	0.478**	9.173**	
EM Europe and Middle East	0.333*	0.002	0.375**	0.448**	0.483**	0.407**	0.411**	8.406**
Bull/Low Volatility State								
Pacific EX JP	4.095**							
JP	0.478**	5.376**						
Europe EX UK	0.706**	0.429**	4.244**					
UK	0.712**	0.443**	0.727**	4.281**				
North America	0.665**	0.372**	0.713**	0.767**	3.681**			
EM Latin America	0.574**	0.276*	0.620**	0.427**	0.537**	6.888**		
EM Asia	0.803**	0.456**	0.600**	0.534**	0.565**	0.509**	5.615**	
EM Europe and Middle East	0.512**	0.386**	0.532**	0.267*	0.335*	0.554**	0.554**	7.176**
3. Transition probabilities	Bear/High Volatility State				Bull/Low Volatility State			
Bear/High Volatility State	0.807**				0.193			
Bull/Low Volatility State	0.103				0.897**			

Panel C - MARKOV CHAIN PROPERTIES, TWO-STATE MODEL

	Bear	Bull			Bear	Bull
Ergodic Probs	0.348	0.652		**Average duration (in months)**	5.18	9.70

** Statistical significance at 1%; * Statistical significance at 5%.

Table 4 (continued)

Estimated Markov Switching Models

Panel B (CRSP Industry Returns, 1926:07 - 2008:07)

	Non-Durables	Durables	Manufacture	Energy	Hi Tech	Telecom	Shops/ Distrib.	Health	Utilities	Other
SINGLE STATE MODEL										
1. Mean returns	0.978**	1.074**	1.034**	1.097**	1.094**	0.831**	0.975**	1.089**	0.902**	0.921**
2. Correlations/Volatilities										
Non-Durables	4.691**									
Durables	0.754**	7.593**								
Manufacture	0.851**	0.873**	6.322**							
Energy	0.616**	0.607**	0.723**	5.983**						
Hi Tech	0.735**	0.779**	0.862**	0.609**	7.437**					
Telecom	0.671**	0.618**	0.671**	0.495**	0.676**	4.594**				
Shops/ Distrib.	0.866**	0.798**	0.841**	0.576**	0.785**	0.670**	5.884**			
Health	0.801**	0.649**	0.762**	0.562**	0.723**	0.600**	0.740**	5.766**		
Utilities	0.707**	0.635**	0.703**	0.617**	0.624**	0.635**	0.655**	0.625**	5.685**	
Other	0.847**	0.802**	0.905**	0.689**	0.798**	0.695**	0.824**	0.741**	0.740**	6.473**

	Non-Durables	Durables	Manufacture	Energy	Hi Tech	Telecom	Shops/ Distrib.	Health	Utilities	Other
TWO STATE MODEL MSIH(2,0)										
1. Mean returns										
Bear/High Volatility State	0.162	0.751	0.718	0.760	0.650	0.238	0.081	0.483	0.447	0.111
Bull/Low Volatility State	1.212**	1.167**	1.125**	1.194**	1.221**	1.001**	1.232**	1.264**	1.032**	1.154**
2. Correlations/Volatilities										
Bear/High Volatility State										
NoDur	7.233**									
Durbl	0.666**	13.181**								
Manuf	0.810**	0.810**	10.848**							
Enrgy	0.473**	0.492**	0.624**	9.318**						
HiTec	0.695**	0.711**	0.837**	0.490**	12.657**					
Telcm	0.653**	0.511**	0.590**	0.393*	0.488**	7.395**				
Shops	0.832**	0.718**	0.796**	0.417**	0.732**	0.591**	9.543**			
Hlth	0.775**	0.543**	0.739**	0.434**	0.686**	0.522**	0.682**	8.903**		
Utils	0.671**	0.499**	0.600**	0.532**	0.470**	0.621**	0.528**	0.514**	10.021**	
Other	0.832**	0.746**	0.890**	0.605**	0.770**	0.625**	0.801**	0.706**	0.662**	10.987**
Bull/Low Volatility State										
NoDur	3.607**									
Durbl	0.826**	4.913**								
Manuf	0.893**	0.905**	4.186**							
Enrgy	0.739**	0.689**	0.797**	4.586**						
HiTec	0.772**	0.814**	0.875**	0.691**	4.999**					
Telcm	0.683**	0.686**	0.724**	0.574**	0.795**	3.360**				
Shops	0.894**	0.850**	0.872**	0.698**	0.821**	0.723**	4.247**			
Hlth	0.823**	0.729**	0.787**	0.670**	0.756**	0.660**	0.785**	4.453**		
Utils	0.744**	0.698**	0.754**	0.681**	0.700**	0.649**	0.729**	0.706**	3.550**	
Other	0.866**	0.832**	0.914**	0.752**	0.814**	0.738**	0.838**	0.771**	0.778**	4.351**

3. Transition probabilities	Bear/High Volatility State	Bull/Low Volatility State
Bear/High Volatility State	0.822**	0.178
Bull/Low Volatility State	0.051	0.949**

Panel C - MARKOV CHAIN PROPERTIES, TWO-STATE MODEL

	Bear	Bull		Bear	Bull
Ergodic Probs	0.224	0.776	**Average duration (in months)**	5.60	19..46

** Statistical significance at 1%; * Statistical significance at 5%.

Table 4 (continued)

Estimated Markov Switching Models

Panel C (International Book-to-Market Sorted Portfolio Local Returns, 1975:01 - 2007:12)

	World	EU ex-UK ex-Scand Value	EU ex-UK ex-Scand Growth	UK Value	UK Growth	Asia & Pacific Value	Asia & Pacific Growth	Scandinavia Value	Scandinavia Growth	US Value	US Growth
SINGLE STATE MODEL											
1. Mean returns	0.694**	1.326**	0.967**	1.725**	1.353**	1.300**	0.401	1.676**	1.486**	1.081**	1.448**
2. Correlations/Volatilities											
World	3.844**										
EU ex-UK ex-Scand Value	0.742**	4.740**									
EU ex-UK ex-Scand Growth	0.792**	0.850**	4.294**								
United Kingdom Value	0.632**	0.573**	0.549**	6.187**							
United Kingdom Growth	0.644**	0.506**	0.584**	0.786**	5.851**						
Asia & Pacific Value	0.593**	0.470**	0.434**	0.403**	0.317**	5.123**					
Asia & Pacific Growth	0.681**	0.441**	0.513**	0.389**	0.357**	0.649**	4.936**				
Scandinavia Value	0.525**	0.584**	0.500**	0.419**	0.370**	0.412**	0.344**	6.428**			
Scandinavia Growth	0.625**	0.542**	0.619**	0.377**	0.407**	0.332**	0.454**	0.643**	6.270**		
United States Value	0.859**	0.532**	0.621**	0.464**	0.550**	0.337**	0.409**	0.372**	0.546**	4.769**	
United States Growth	0.804**	0.623**	0.593**	0.556**	0.563**	0.360**	0.350**	0.432**	0.434**	0.784**	4.302**
TWO STATE MODEL MSIH(2,0)											
	World	EU ex-UK ex-Scand Value	EU ex-UK ex-Scand Growth	UK Value	UK Growth	Asia & Pacific Value	Asia & Pacific Growth	Scandinavia Value	Scandinavia Growth	US Value	US Growth
1. Mean returns											
Regime 1 (Bull Word/Low Vol.)	0.956**	1.851**	1.446**	1.512**	1.296**	1.168**	0.612*	1.690**	1.392**	1.317**	1.557**
Regime 2 (Bear Word/High Vol	0.049	0.036	-0.209	2.247*	1.494	1.623*	-0.118	1.643*	1.716	0.501	1.182
2. Correlations/Volatilities											
Regime 1 (Bull Word/Low Vol.)											
World	2.910**										
EU ex-UK ex-Scand Value	0.714**	4.128**									
EU ex-UK ex-Scand Growth	0.787**	0.860**	3.268**								
United Kingdom Value	0.613**	0.562**	0.557**	4.431**							
United Kingdom Growth	0.636**	0.490**	0.552**	0.760**	3.961**						
Asia & Pacific Value	0.605**	0.432**	0.466**	0.335*	0.306*	3.886**					
Asia & Pacific Growth	0.720**	0.425**	0.501**	0.439**	0.410**	0.718**	3.997**				
Scandinavia Value	0.507**	0.532**	0.466**	0.496**	0.431**	0.383*	0.394**	5.387**			
Scandinavia Growth	0.595**	0.495**	0.518**	0.446**	0.457*	0.282*	0.385**	0.651**	4.740**		
United States Value	0.844**	0.454**	0.574**	0.389**	0.493**	0.325**	0.459**	0.319*	0.510**	3.664**	
United States Growth	0.817**	0.614**	0.626**	0.518**	0.517**	0.327**	0.420**	0.403**	0.507**	0.786**	3.133**
Regime 2 (Bear Word/High Vol.)											
World	5.438**										
EU ex-UK ex-Scand Value	0.777**	5.767**									
EU ex-UK ex-Scand Growth	0.791**	0.845**	5.948**								
United Kingdom Value	0.663**	0.637**	0.576**	9.131**							
United Kingdom Growth	0.661**	0.563**	0.628**	0.801**	8.915**						
Asia & Pacific Value	0.598**	0.544**	0.435**	0.446**	0.324*	7.297**					
Asia & Pacific Growth	0.648**	0.454**	0.518**	0.365*	0.331*	0.605**	6.663**				
Scandinavia Value	0.550**	0.663**	0.543**	0.369**	0.338**	0.440**	0.298*	8.436**			
Scandinavia Growth	0.656**	0.622**	0.716**	0.333*	0.380*	0.365**	0.514**	0.642**	8.959**		
United States Value	0.868**	0.609**	0.651**	0.524**	0.587**	0.353*	0.363*	0.422**	0.577**	6.705**	
United States Growth	0.797**	0.663**	0.577**	0.584**	0.591**	0.386*	0.299*	0.460**	0.389**	0.785**	6.293**

3. Transition probabilities	Regime 1 (Bull Word/Low Volatility)	Regime 2 (Bear Word/High Volatility)
Regime 1 (Bull Word/Low Vol.)	0.901**	0.099
Regime 2 (Bear Word/High Vol	0.292	0.708**

Panel C - MARKOV CHAIN PROPERTIES, TWO-STATE MODEL

	Regime 1	Regime 2		Regime 1	Regime 2
Ergodic Probs	0.747	0.253	**Average duration (in months)**	10.10	3.43

** Statistical significance at 1%; * Statistical significance at 5%.

Table 5

Moments Implied by Estimated Two-State Markov Switching Model

This table reports moment implied by the estimated Two-State Model for returns. In the co-skewness matrix, coefficients above the main diagonal refer to the sample covariance between the square of the returns of the row portfolio/index and the level of returns of the column portfolio/index; coefficients below the main diagonal refer to the sample covariance between the level of the returns of the column portfolio/index and the square of returns of the row portfolio/index. In the correlation/co-kurtosis matrix, correlations are reported above the main diagonal and sample covariances between squared portfolio returns appear below the main diagonal. Panels A, B and C respectively refer to the International, the Industry, and the International Book-to-Market portfolios.

Panel A (International MSCI USD Returns, 1988:01 - 2008:08)

	Mean	St. Dev.	Sharpe ratio	Median	Min.	Max.	Skewness	Kurtosis
Pacific ex-Japan	0.713	4.381	0.082	0.830	-21.8	17.8	-0.308	4.222
Japan	-0.060	5.084	-0.081	0.037	-12.1	12.3	-0.058	3.592
Europe ex-UK	0.748	3.964	0.100	0.818	-13.1	12.2	-0.247	3.672
United Kingdom	0.703	3.558	0.098	0.723	-9.8	13.4	-0.005	3.279
North America	0.719	3.192	0.115	0.708	-9.5	9.1	-0.135	3.442
EM Latin America	1.889	7.237	0.212	2.066	-27.3	21.9	-0.286	4.106
EM Asia	0.708	5.747	0.062	0.771	-19.7	17.0	-0.100	3.789
EM Europe & Middle East	1.843	6.290	0.237	1.875	-22.1	22.0	-0.055	4.101

Correlation and (variance) co-kurtosis matrices

	Pacific ex-JP	Japan	EU ex-UK	UK	North Amr.	EM Latin Amr.	EM Asia	EM EU & Middle East
Pacific ex-JP		0.447	0.594	0.621	0.603	0.545	0.784	0.426
Japan	1.457		0.459	0.476	0.369	0.318	0.416	0.223
EU ex-UK	1.873	1.635		0.740	0.671	0.417	0.513	0.469
UK	1.731	1.494	2.241		0.666	0.397	0.445	0.353
North Amr.	1.826	1.313	2.333	2.001		0.501	0.552	0.401
EM Latin Amr.	1.862	1.350	1.788	1.322	1.861		0.492	0.480
EM Asia	2.833	1.671	1.960	1.430	1.799	1.716		0.481
EM EU & Middle East	1.428	1.253	1.665	1.272	1.732	1.914	1.535	

Co-skewness matrix

	Pacific ex-JP	Japan	EU ex-UK	UK	North Amr.	EM Latin Amr.	EM Asia	EM EU & Middle East
Pacific ex-JP		-0.164	-0.195	-0.124	-0.178	-0.210	-0.213	-0.166
Japan	-0.104		-0.062	-0.008	-0.054	-0.152	-0.084	-0.048
EU ex-UK	-0.193	-0.168		-0.181	-0.227	-0.164	-0.212	-0.172
UK	-0.030	-0.055	-0.092		-0.021	-0.055	-0.048	-0.090
North Amr.	-0.172	-0.107	-0.219	-0.093		-0.203	-0.175	-0.186
EM Latin Amr.	-0.218	-0.215	-0.112	-0.144	-0.180		-0.135	-0.179
EM Asia	-0.170	-0.142	-0.182	-0.111	-0.136	-0.142		-0.154
EM EU & Middle East	-0.103	-0.045	-0.068	-0.114	-0.163	-0.247	-0.114	

Table 5 (continued)

Moments Implied by Estimated Two-State Markov Switching Model

Panel B (CRSP Industry Returns, 1926:07 - 2008:07)

	Mean	Std. Dev.	Sharpe ratio	Median	Min.	Max.	Skewness	Kurtosis
Non-Durables	0.969	4.198	0.158	1.050	-20.7	21.0	-0.147	6.187
Durables	1.056	6.759	0.111	1.104	-31.2	40.6	0.338	10.426
Manufacture	1.017	5.631	0.126	1.074	-24.8	33.3	0.263	9.314
Energy	1.095	5.369	0.147	1.147	-18.5	27.5	0.034	5.315
Hi Tech	1.086	6.624	0.118	1.139	-27.0	37.3	0.036	7.112
Telecom	0.827	4.114	0.127	0.908	-11.0	18.4	-0.147	5.761
Shops/ Distrib.	0.965	5.243	0.126	1.047	-25.8	26.0	-0.126	6.472
Health	1.069	5.149	0.148	1.138	-23.8	24.0	-0.037	6.688
Utilities	0.891	5.113	0.115	0.959	-15.8	24.6	-0.093	8.157
Other	0.905	5.792	0.104	1.003	-19.6	31.5	0.174	9.673

Correlation and (variance) co-kurtosis matrices

	Non-Durables	Durables	Manuf.	Energy	Hi Tech	Telecom	Shops/ Distrib.	Health	Utilities	Other
Non-Durables		0.754	0.849	0.616	0.732	0.671	0.866	0.803	0.709	0.846
Durables	6.005		0.873	0.610	0.778	0.619	0.799	0.652	0.636	0.802
Manufacture	6.145	8.597		0.728	0.861	0.673	0.840	0.760	0.706	0.904
Energy	3.664	4.456	4.849		0.609	0.501	0.577	0.562	0.616	0.690
Hi Tech	4.869	6.595	6.749	3.806		0.677	0.785	0.721	0.627	0.797
Telecom	3.561	4.013	4.341	2.744	4.311		0.672	0.600	0.636	0.695
Shops/ Distrib.	5.285	6.052	6.130	3.530	5.211	3.849		0.744	0.660	0.823
Health	5.008	5.526	5.804	3.432	4.925	3.582	4.909		0.629	0.739
Utilities	4.607	5.344	5.831	3.696	5.077	4.230	4.999	4.822		0.740
Other	5.535	6.608	7.628	4.405	6.082	4.707	5.749	5.651	6.344	

Co-skewness matrix

	Non-Durables	Durables	Manuf.	Energy	Hi Tech	Telecom	Shops/ Distrib.	Health	Utilities	Other
Non-Durables		-0.013	-0.036	-0.092	-0.069	-0.147	-0.132	-0.101	-0.129	-0.101
Durables	0.126		0.294	0.145	0.199	-0.025	0.115	0.143	0.063	0.144
Manufacture	0.082	0.274		0.111	0.174	0.006	0.082	0.136	0.086	0.178
Energy	-0.067	0.046	0.040		0.005	-0.071	-0.081	-0.012	-0.039	0.003
Hi Tech	-0.023	0.099	0.099	0.015		-0.089	-0.016	0.014	-0.021	0.037
Telecom	-0.143	-0.129	-0.087	-0.115	-0.119		-0.147	-0.107	-0.087	-0.085
Shops/ Distrib.	-0.127	-0.010	-0.025	-0.092	-0.060	-0.135		-0.103	-0.104	-0.067
Health	-0.066	0.042	0.055	-0.010	-0.004	-0.066	-0.071		-0.064	0.021
Utilities	-0.128	-0.039	-0.007	-0.078	-0.053	-0.079	-0.114	-0.091		-0.014
Other	-0.011	0.125	0.165	0.048	0.095	0.010	-0.122	0.088	0.077	

Table 5 (continued)

Moments Implied by Estimated Two-State Markov Switching Model

Panel C (International Book-to-Market Sorted Portfolio Local Returns, 1975:01 - 2007:12)

	Mean	St. Dev.	Sharpe ratio	Median	Min.	Max.	Skewness	Kurtosis
World	0.717	3.244	0.072	0.764	-18.2	14.6	-0.440	5.207
EU ex-UK ex-Scand Value	1.391	4.055	0.224	1.487	-11.4	15.7	-0.269	4.122
EU ex-UK ex-Scand Growth	1.024	3.619	0.149	1.131	-15.2	15.9	-0.441	5.205
United Kingdom Value	1.689	5.171	0.233	1.609	-19.4	36.0	0.385	6.919
United Kingdom Growth	1.335	4.870	0.175	1.300	-22.0	42.3	0.591	10.34
Asia & Pacific Value	1.285	4.338	0.184	1.253	-13.0	16.0	0.040	4.894
Asia & Pacific Growth	0.405	4.172	-0.019	0.441	-14.5	22.0	-0.060	4.418
Scandinavia Value	1.680	5.505	0.217	1.653	-12.5	24.6	0.065	4.042
Scandinavia Growth	1.443	5.315	0.180	1.415	-21.9	26.6	0.048	4.641
United States Value	1.091	4.028	0.151	1.143	-22.4	16.3	-0.247	4.549
United States Growth	1.447	3.606	0.267	1.456	-15.7	21.4	-0.086	5.689

Correlation and (variance) co-kurtosis matrices

	World	EU ex-UK ex-Scand Value	EU ex-UK ex-Scand Growth	UK Value	UK Growth	Asia Pacific Value	Asia Pacific Growth	Scandinavia Value	Scandinavia Growth	United States Value	United States Growth
World		0.741	0.792	0.626	0.634	0.601	0.683	0.528	0.626	0.857	0.803
EU ex-UK ex-Scand Value	3.218		0.851	0.567	0.496	0.478	0.442	0.580	0.538	0.524	0.623
EU ex-UK ex-Scand Growth	4.030	3.488		0.543	0.574	0.447	0.513	0.498	0.613	0.617	0.594
United Kingdom Value	3.324	2.427	3.011		0.779	0.399	0.386	0.424	0.379	0.453	0.548
United Kingdom Growth	3.633	2.402	3.477	6.853		0.315	0.350	0.370	0.408	0.535	0.552
Asia & Pacific Value	3.021	2.237	2.677	2.357	2.170		0.653	0.416	0.337	0.345	0.365
Asia & Pacific Growth	2.599	1.742	2.217	1.740	1.716	2.572		0.352	0.454	0.413	0.357
Scandinavia Value	2.245	2.137	2.204	1.833	1.822	1.902	1.521		0.647	0.371	0.431
Scandinavia Growth	2.749	2.118	2.866	1.883	2.034	1.864	1.920	2.254		0.544	0.442
United States Value	3.886	2.328	3.088	2.513	2.836	2.128	1.743	1.820	2.403		0.783
United States Growth	3.964	2.700	3.165	3.677	4.441	2.220	1.645	1.936	1.993	4.824	

Co-Skewness matrices

	World	EU ex-UK ex-Scand Value	EU ex-UK ex-Scand Growth	UK Value	UK Growth	Asia Pacific Value	Asia Pacific Growth	Scandinavia Value	Scandinavia Growth	United States Value	United States Growth
World		-0.382	-0.439	-0.193	-0.205	-0.295	-0.250	-0.244	-0.261	-0.350	-0.306
EU ex-UK ex-Scand Value	-0.316		-0.306	-0.177	-0.195	-0.215	-0.162	-0.204	-0.205	-0.268	-0.260
EU ex-UK ex-Scand Growth	-0.425	-0.363		-0.228	-0.247	-0.291	-0.232	-0.270	-0.273	-0.367	-0.326
United Kingdom Value	-0.018	-0.084	-0.078		0.397	0.014	-0.025	-0.068	-0.044	-0.034	0.137
United Kingdom Growth	-0.072	-0.078	-0.049	0.470		-0.022	-0.031	-0.042	0.004	-0.016	0.203
Asia & Pacific Value	-0.204	-0.179	-0.222	-0.044	-0.109		-0.076	-0.125	-0.122	-0.186	-0.168
Asia & Pacific Growth	-0.153	-0.132	-0.141	-0.030	-0.055	-0.055		-0.108	-0.116	-0.122	-0.108
Scandinavia Value	-0.135	-0.147	-0.160	-0.049	-0.082	-0.059	-0.050		-0.023	-0.119	-0.105
Scandinavia Growth	-0.147	-0.166	-0.162	-0.070	-0.059	-0.105	-0.085	-0.027		-0.086	-0.107
United States Value	-0.295	-0.298	-0.356	-0.130	-0.165	-0.193	-0.158	-0.163	-0.164		-0.202
United States Growth	-0.216	-0.266	-0.298	0.025	0.004	-0.150	-0.139	-0.151	-0.134	-0.179	

Table 6

Portfolio Weights as a Function of the Initial State

This table displays average optimal portfolio shares. The out-of-sample period for our International (Panel A), Industry (Panel B) and Book-to-Market International (Panel C) data runs from 1998:01-2008:07, 1980:01-2008:07, and 1995:01-2007:12 respectively. The LHS (RHS) refers to portfolios subject (free) from short-sales constraints. The first six columns refer to the investor horizon. The upper (lower) part of each panel refers to the allocation associated with the single-state (two-state) model. In the latter case, we highlight the "ex-ante" portfolio shares computed using the ergodic probabilities, and the shares conditional on the bear and the bull states. Each row, within a given case, is associated with investor preferences ranging from mean-variance to four-moments.

Panel A (International MSCI USD Returns, 1988:01 - 2008:08)

Preferences		T=1	T=3	T=12	T=24	T=60	T=120	"Slope"	T=1	T=3	T=12	T=24	T=60	T=120	"Slope"
		No-short sales							Unconstrained						
		Single-State Model (Unconditional Allocation)													
	Pacific EX JP	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.108	-0.136	-0.096	-0.072	-0.116	-0.222	-0.114
	Japan	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.948	-0.969	-0.994	-1.067	-1.216	-1.535	-0.587
	Europe EX UK	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.084	-0.017	-0.037	-0.049	-0.057	-0.034	0.050
	UK	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.631	0.608	0.613	0.643	0.679	0.793	0.163
	North America	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.316	0.304	0.307	0.279	0.263	0.143	-0.173
	EM Latin America	0.383	0.391	0.398	0.400	0.407	0.415	0.033	0.642	0.683	0.676	0.706	0.810	1.038	0.397
	EM Asia	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.313	-0.309	-0.329	-0.343	-0.381	-0.418	-0.105
	EM Europe and Middle East	0.617	0.609	0.602	0.600	0.593	0.585	-0.033	0.865	0.836	0.861	0.902	1.018	1.235	0.370
		Two-State Model (Current State: Ergodic/Unconditional Probabilities)													
Mean-Variance Preferences	Pacific EX JP	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.787	-0.562	-0.870	-0.923	-1.052	-1.318	-2.105
	Japan	0.162	0.000	0.000	0.000	0.000	0.000	-0.162	-1.306	-1.219	-1.230	-1.285	-1.482	-1.848	-0.542
	Europe EX UK	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.426	-0.207	-0.127	-0.088	-0.104	-0.104	1.323
	UK	0.122	0.000	0.000	0.000	0.000	0.000	-0.122	2.063	1.133	1.071	1.099	1.195	1.358	-0.705
	North America	0.413	0.306	0.288	0.276	0.260	0.243	-0.170	-0.087	0.768	1.058	1.054	1.146	1.246	1.333
	EM Latin America	0.134	0.289	0.291	0.289	0.305	0.331	0.197	1.148	0.588	0.540	0.554	0.644	0.802	-0.346
	EM Asia	0.018	0.000	0.000	0.000	0.000	0.000	-0.018	-1.007	0.077	0.320	0.346	0.410	0.520	1.527
	EM Europe and Middle East	0.150	0.405	0.421	0.435	0.435	0.426	0.276	0.828	0.423	0.238	0.243	0.243	0.344	-0.485
Mean-Var-Skew Preferences	Pacific EX JP	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-4.185	-2.865	-2.401	-1.895	-1.162	-0.695	3.490
	Japan	0.165	0.000	0.000	0.000	0.000	0.000	-0.165	-3.622	-2.672	-2.065	-1.357	-1.625	-1.492	2.130
	Europe EX UK	0.000	0.000	0.000	0.000	0.000	0.000	0.000	5.864	3.288	1.839	-0.182	-0.104	-0.123	-5.987
	UK	0.127	0.000	0.000	0.000	0.000	0.000	-0.127	-3.740	-1.240	-0.275	0.940	1.063	1.144	4.884
	North America	0.415	0.349	0.320	0.277	0.265	0.239	-0.175	4.789	3.219	2.053	1.407	1.387	1.115	-3.675
	EM Latin America	0.123	0.217	0.274	0.300	0.304	0.325	0.202	1.415	0.831	1.323	0.841	0.752	0.870	-0.546
	EM Asia	0.023	0.000	0.000	0.000	0.000	0.000	-0.023	-4.385	-2.241	-1.468	-0.613	0.430	0.587	4.972
	EM Europe and Middle East	0.147	0.434	0.406	0.423	0.432	0.436	0.289	4.864	2.680	1.994	1.857	0.259	-0.405	-5.269
Mean-Var-Skew-Kurtosis	Pacific EX JP	0.010	0.001	0.000	0.000	0.000	0.000	-0.010	-0.071	-0.019	-0.081	-0.153	-0.316	-0.560	-0.489
	Japan	0.223	0.204	0.098	0.000	0.000	0.000	-0.223	0.286	0.254	0.112	-0.026	-0.333	-0.735	-1.021
	Europe EX UK	0.017	0.022	0.044	0.031	0.000	0.000	-0.017	-0.052	0.035	0.055	0.044	0.052	0.031	0.083
	UK	0.130	0.088	0.111	0.138	0.023	0.000	-0.130	0.173	0.096	0.163	0.235	0.374	0.587	0.414
	North America	0.244	0.258	0.249	0.253	0.269	0.225	-0.019	0.087	0.092	0.124	0.169	0.277	0.407	0.320
	EM Latin America	0.148	0.151	0.218	0.263	0.316	0.342	0.194	0.207	0.180	0.252	0.301	0.428	0.595	0.389
	EM Asia	0.033	0.061	0.030	0.019	0.003	0.000	-0.033	0.090	0.088	0.097	0.135	0.185	0.284	0.194
	EM Europe and Middle East	0.195	0.215	0.250	0.297	0.388	0.433	0.238	0.280	0.274	0.277	0.296	0.331	0.390	0.111
Mean-Var-Kurtosis Preferences	Pacific EX JP	0.013	0.002	0.000	0.000	0.000	0.000	-0.013	-0.079	-0.018	-0.082	-0.153	-0.313	-0.540	-0.461
	Japan	0.247	0.225	0.099	0.000	0.000	0.000	-0.247	0.272	0.244	0.108	-0.032	-0.336	-0.740	-1.012
	Europe EX UK	0.028	0.032	0.050	0.043	0.000	0.000	-0.028	-0.035	0.031	0.054	0.055	0.055	0.044	0.078
	UK	0.125	0.098	0.120	0.147	0.034	0.000	-0.125	0.163	0.116	0.160	0.228	0.377	0.585	0.423
	North America	0.153	0.158	0.180	0.204	0.240	0.214	0.061	0.081	0.081	0.123	0.162	0.269	0.393	0.312
	EM Latin America	0.174	0.172	0.230	0.270	0.316	0.344	0.170	0.221	0.192	0.250	0.304	0.430	0.594	0.373
	EM Asia	0.036	0.070	0.058	0.029	0.006	0.000	-0.036	0.088	0.088	0.119	0.141	0.188	0.284	0.195
	EM Europe and Middle East	0.225	0.243	0.263	0.307	0.404	0.442	0.217	0.288	0.266	0.269	0.296	0.330	0.381	0.092
		Two-State Model (Current State: Bear/High Volatility)													
Mean-Variance Preferences	Pacific EX JP	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-2.070	-1.678	-1.625	-1.715	-1.898	-2.350	-0.280
	Japan	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.822	-1.574	-1.535	-1.616	-1.832	-2.273	-0.451
	Europe EX UK	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.775	-0.464	-0.382	-0.351	-0.389	-0.389	0.387
	UK	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2.642	2.024	1.811	1.878	1.997	2.319	-0.323
	North America	0.997	0.867	0.820	0.789	0.747	0.690	-0.307	2.397	1.866	1.859	1.857	2.033	2.319	-0.078
	EM Latin America	0.003	0.072	0.090	0.097	0.125	0.160	0.157	0.338	0.374	0.393	0.423	0.478	0.620	0.282
	EM Asia	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.298	1.013	0.948	0.992	1.118	1.374	0.076
	EM Europe and Middle East	0.000	0.061	0.090	0.114	0.129	0.150	0.150	-1.008	-0.560	-0.469	-0.467	-0.507	-0.620	0.388
Mean-Var-Skew Preferences	Pacific EX JP	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-3.501	-2.482	-2.863	-3.110	-2.280	-1.368	2.133
	Japan	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-4.092	-2.740	-1.698	-1.035	-2.264	-2.881	1.211
	Europe EX UK	0.000	0.000	0.000	0.000	0.000	0.000	0.000	6.481	5.422	2.221	-1.780	-0.348	-0.486	-6.967
	UK	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.503	-0.642	1.259	4.216	2.069	1.656	2.159
	North America	1.000	1.000	0.907	0.788	0.751	0.686	-0.314	5.573	4.342	2.690	2.292	2.156	1.904	-3.668
	EM Latin America	0.000	0.000	0.055	0.095	0.115	0.166	0.166	-5.916	-3.440	0.525	0.442	0.758	0.781	6.697
	EM Asia	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-2.099	-0.608	-0.226	0.202	1.419	1.623	3.722
	EM Europe and Middle East	0.000	0.000	0.038	0.117	0.134	0.148	0.148	5.058	1.148	-0.908	-0.226	-0.509	-0.230	-5.288

Panel A (International MSCI USD Returns, 1988:01 - 2008:08)

		T=1	T=3	T=12	T=24	T=60	T=120	"Slope"	T=1	T=3	T=12	T=24	T=60	T=120	"Slope"
		Two-State Model (Current State: Bear/High Volatility)													
		No-short sales							**Unconstrained**						
Mean-Var-Skew-Kurtosis	Pacific EX JP	0.028	0.002	0.000	0.000	0.000	0.000	-0.028	0.083	0.003	-0.136	-0.278	-0.570	-0.982	-1.065
	Japan	0.155	0.135	0.039	0.000	0.000	0.000	-0.155	0.324	0.270	0.077	-0.071	-0.403	-0.844	-1.168
	Europe EX UK	0.048	0.062	0.048	0.027	0.000	0.000	-0.048	0.139	0.125	0.087	0.064	0.025	-0.033	-0.171
	UK	0.000	0.000	0.057	0.078	0.024	0.000	0.000	-0.075	-0.002	0.165	0.292	0.565	0.920	0.995
	North America	0.511	0.527	0.534	0.527	0.625	0.638	0.127	0.012	0.053	0.174	0.282	0.495	0.794	0.782
	EM Latin America	0.068	0.078	0.130	0.144	0.153	0.181	0.113	0.140	0.157	0.221	0.242	0.327	0.457	0.317
	EM Asia	0.029	0.047	0.053	0.053	0.010	0.000	-0.029	0.035	0.094	0.188	0.294	0.485	0.739	0.705
	EM Europe and Middle East	0.162	0.150	0.140	0.172	0.188	0.182	0.020	0.343	0.300	0.225	0.175	0.076	-0.052	-0.394
Mean-Var-Kurtosis Preferences	Pacific EX JP	0.037	0.007	0.000	0.000	0.000	0.000	-0.037	0.078	0.010	-0.141	-0.278	-0.573	-0.977	-1.055
	Japan	0.226	0.193	0.060	0.000	0.000	0.000	-0.226	0.316	0.250	0.079	-0.084	-0.411	-0.860	-1.175
	Europe EX UK	0.080	0.090	0.072	0.039	0.000	0.000	-0.080	0.165	0.117	0.090	0.072	0.032	-0.014	-0.179
	UK	0.000	0.006	0.086	0.117	0.033	0.000	0.000	-0.092	0.014	0.166	0.295	0.566	0.919	1.011
	North America	0.261	0.294	0.342	0.392	0.561	0.614	0.353	0.002	0.063	0.161	0.271	0.492	0.787	0.785
	EM Latin America	0.110	0.123	0.164	0.171	0.175	0.188	0.078	0.158	0.168	0.215	0.251	0.339	0.451	0.293
	EM Asia	0.048	0.070	0.083	0.081	0.016	0.000	-0.048	0.036	0.094	0.201	0.300	0.485	0.747	0.711
	EM Europe and Middle East	0.239	0.217	0.193	0.199	0.215	0.197	-0.041	0.338	0.284	0.229	0.174	0.070	-0.053	-0.391
		Two-State Model (Current State: Bull/Low Volatility)													
Mean-Variance Preferences	Pacific EX JP	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2.225	0.040	-0.457	-0.472	-0.583	-0.744	-2.969
	Japan	0.247	0.000	0.000	0.000	0.000	0.000	-0.247	-1.017	-0.969	-1.040	-1.093	-1.270	-1.577	-0.560
	Europe EX UK	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.750	-0.065	0.013	0.053	0.051	0.048	1.798
	UK	0.187	0.000	0.000	0.000	0.000	0.000	-0.187	1.710	0.618	0.663	0.643	0.734	0.806	-0.904
	North America	0.093	0.000	0.000	0.000	0.000	0.000	-0.093	-1.395	0.158	0.615	0.604	0.638	0.661	2.057
	EM Latin America	0.201	0.395	0.382	0.383	0.395	0.402	0.201	1.562	0.695	0.588	0.615	0.704	0.878	-0.684
	EM Asia	0.028	0.000	0.000	0.000	0.000	0.000	-0.028	-2.214	-0.414	-0.016	-0.003	0.030	0.061	2.276
	EM Europe and Middle East	0.245	0.605	0.618	0.617	0.605	0.598	0.353	1.880	0.936	0.633	0.653	0.697	0.866	-1.014
Mean-Var-Skew Preferences	Pacific EX JP	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-4.352	-2.954	-2.096	-1.166	-0.537	-0.315	4.037
	Japan	0.253	0.000	0.000	0.000	0.000	0.000	-0.253	-3.203	-2.596	-2.193	-1.504	-1.246	-0.730	2.473
	Europe EX UK	0.000	0.000	0.000	0.000	0.000	0.000	0.000	5.457	2.034	1.611	0.668	0.025	0.070	-5.388
	UK	0.193	0.000	0.000	0.000	0.000	0.000	-0.193	-5.308	-1.557	-1.084	-0.794	0.514	0.835	6.143
	North America	0.093	0.000	0.000	0.000	0.000	0.000	-0.093	4.182	2.556	1.634	0.904	0.926	0.663	-3.519
	EM Latin America	0.188	0.328	0.390	0.393	0.387	0.395	0.207	5.134	3.128	1.700	1.007	0.721	0.895	-4.239
	EM Asia	0.035	0.000	0.000	0.000	0.000	0.000	-0.035	-5.460	-3.036	-2.031	-1.002	-0.100	0.026	5.486
	EM Europe and Middle East	0.239	0.672	0.610	0.607	0.613	0.605	0.366	4.549	3.425	3.460	2.888	0.698	-0.444	-4.993
Mean-Var-Skew-Kurtosis	Pacific EX JP	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.152	-0.029	-0.050	-0.083	-0.167	-0.322	-0.170
	Japan	0.253	0.240	0.125	0.000	0.000	0.000	-0.253	0.253	0.237	0.124	-0.002	-0.292	-0.642	-0.895
	Europe EX UK	0.000	0.000	0.039	0.031	0.000	0.000	0.000	-0.148	-0.013	0.037	0.032	0.063	0.065	0.213
	UK	0.193	0.130	0.139	0.165	0.022	0.000	-0.193	0.301	0.147	0.157	0.193	0.258	0.383	0.082
	North America	0.093	0.105	0.090	0.100	0.075	0.000	-0.093	0.122	0.112	0.093	0.105	0.157	0.191	0.069
	EM Latin America	0.188	0.187	0.259	0.317	0.392	0.418	0.230	0.230	0.191	0.266	0.327	0.471	0.653	0.424
	EM Asia	0.035	0.066	0.017	0.000	0.000	0.000	-0.035	0.117	0.080	0.046	0.046	0.024	0.037	-0.080
	EM Europe and Middle East	0.239	0.272	0.331	0.387	0.511	0.582	0.343	0.277	0.275	0.328	0.382	0.486	0.634	0.357
Mean-Var-Kurtosis Preferences	Pacific EX JP	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.159	-0.032	-0.049	-0.084	-0.168	-0.301	-0.141
	Japan	0.247	0.235	0.119	0.000	0.000	0.000	-0.247	0.248	0.233	0.118	-0.004	-0.288	-0.647	-0.895
	Europe EX UK	0.000	0.000	0.037	0.042	0.000	0.000	0.000	-0.138	-0.015	0.034	0.044	0.066	0.071	0.209
	UK	0.187	0.144	0.133	0.160	0.034	0.000	-0.187	0.295	0.162	0.150	0.188	0.271	0.387	0.093
	North America	0.093	0.083	0.092	0.099	0.065	0.000	-0.093	0.120	0.090	0.096	0.103	0.143	0.172	0.052
	EM Latin America	0.201	0.194	0.253	0.317	0.389	0.425	0.224	0.242	0.198	0.258	0.328	0.465	0.659	0.416
	EM Asia	0.028	0.066	0.042	0.000	0.000	0.000	-0.028	0.113	0.082	0.071	0.052	0.028	0.030	-0.082
	EM Europe and Middle East	0.245	0.278	0.323	0.381	0.511	0.575	0.330	0.279	0.281	0.321	0.374	0.482	0.628	0.348

Table 6 (continued)

Portfolio Weights as a Function of the Initial State

Panel B (CRSP Industry Returns, 1926:07 - 2008:07)

		T=1	T=3	T=12	T=24	T=60	T=120	"Slope"	T=1	T=3	T=12	T=24	T=60	T=120	"Slope"
		No-short sales							Unconstrained						
		Single-State Model (Unconditional Allocation)													
	Non Durables	0.106	0.000	0.000	0.095	0.159	0.194	0.088	0.868	1.055	0.820	0.846	0.834	0.838	-0.031
	Durables	0.000	0.000	0.000	0.021	0.000	0.000	0.000	0.288	0.446	0.304	0.145	0.039	0.046	-0.242
	Manufacturing	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.154	-0.104	-0.138	-0.134	-0.129	-0.138	0.016
	Energy	0.461	0.503	0.585	0.653	0.596	0.604	0.143	0.699	1.073	0.506	0.404	0.407	0.411	-0.288
	Hi Tech	0.000	0.000	0.145	0.070	0.045	0.040	0.040	0.213	0.459	0.259	0.022	0.029	0.025	-0.188
	Telecommunications	0.000	0.000	0.000	0.000	0.115	0.113	0.113	-0.090	-0.632	-0.140	0.160	0.255	0.309	0.399
	Shops/Distribution	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.172	-0.351	-0.107	-0.110	-0.103	-0.099	0.073
	Health	0.433	0.497	0.177	0.161	0.085	0.049	-0.384	0.544	0.918	0.326	0.274	0.284	0.227	-0.317
	Utilities	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.174	-0.266	-0.080	-0.013	-0.014	-0.014	0.160
	Other	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.023	-1.598	-0.749	-0.593	-0.602	-0.606	0.417
		Two-State Model (Current State: Ergodic/Unconditional Probabilities)													
Mean-Variance Preferences	Non Durables	0.034	0.143	0.359	0.564	0.673	0.678	0.644	0.819	1.104	0.851	0.875	0.857	0.880	0.061
	Durables	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.195	0.468	0.095	0.104	0.106	0.106	-0.089
	Manufacturing	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.139	-0.947	-0.474	-0.516	-0.510	-0.526	-0.387
	Energy	0.463	0.393	0.314	0.290	0.261	0.261	-0.202	0.718	1.095	0.448	0.446	0.443	0.446	-0.271
	Hi Tech	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.189	0.459	0.107	0.116	0.112	0.124	-0.065
	Telecommunications	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.021	-0.857	-0.004	0.133	0.127	0.118	0.140
	Shops/Distribution	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.155	0.059	0.117	0.113	0.128	0.128	0.284
	Health	0.503	0.464	0.327	0.146	0.066	0.061	-0.442	0.612	1.076	0.398	0.251	0.249	0.245	-0.368
	Utilities	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.167	-0.462	-0.047	-0.050	-0.037	-0.050	0.116
	Other	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.050	-0.995	-0.491	-0.471	-0.474	-0.471	0.579
Mean-Var-Skew Preferences	Non Durables	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.326	0.616	0.804	1.016	0.977	0.986	0.660
	Durables	0.046	0.000	0.000	0.000	0.000	0.000	-0.046	0.319	0.440	0.109	0.106	0.110	0.103	-0.216
	Manufacturing	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.252	-0.753	-0.538	-0.560	-0.581	-0.621	-0.369
	Energy	0.954	0.709	0.581	0.345	0.314	0.305	-0.649	0.670	0.816	0.483	0.407	0.445	0.536	-0.134
	Hi Tech	0.000	0.038	0.145	0.290	0.345	0.367	0.367	0.117	0.307	0.126	0.105	0.091	0.106	-0.011
	Telecommunications	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.181	-0.381	0.083	0.151	0.131	0.092	-0.089
	Shops/Distribution	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.076	0.022	0.092	0.117	0.116	0.143	0.219
	Health	0.000	0.253	0.274	0.365	0.341	0.328	0.328	0.328	0.626	0.486	0.287	0.225	0.173	-0.155
	Utilities	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.022	-0.123	-0.016	-0.043	-0.052	-0.045	-0.023
	Other	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.591	-0.569	-0.630	-0.587	-0.461	-0.473	0.118
Mean-Var-Kurtosis Preferences	Non Durables	0.054	0.000	0.145	0.356	0.679	0.686	0.632	0.255	0.456	0.879	0.884	0.867	0.871	0.616
	Durables	0.000	0.053	0.000	0.000	0.000	0.000	0.000	0.264	0.224	0.104	0.095	0.110	0.107	-0.157
	Manufacturing	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.619	-0.420	-0.475	-0.504	-0.524	-0.519	0.100
	Energy	0.358	0.358	0.335	0.300	0.259	0.260	-0.097	0.470	0.461	0.440	0.446	0.445	0.441	-0.029
	Hi Tech	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.089	0.165	0.110	0.120	0.122	0.119	0.030
	Telecommunications	0.297	0.210	0.175	0.098	0.000	0.000	-0.297	0.294	0.201	0.138	0.139	0.125	0.125	-0.169
	Shops/Distribution	0.000	0.047	0.040	0.024	0.000	0.000	0.000	0.029	0.030	0.097	0.108	0.128	0.132	0.102
	Health	0.272	0.323	0.305	0.222	0.062	0.054	-0.218	0.272	0.171	0.227	0.235	0.248	0.244	-0.027
	Utilities	0.019	0.009	0.000	0.000	0.000	0.000	-0.019	0.057	0.014	-0.031	-0.051	-0.044	-0.043	-0.101
	Other	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.111	-0.300	-0.489	-0.472	-0.477	-0.477	-0.366
Mean-Var-Skew-Kurtosis	Non Durables	0.000	0.000	0.144	0.405	0.663	0.679	0.679	0.094	0.156	0.460	0.886	0.873	0.859	0.766
	Durables	0.046	0.079	0.032	0.000	0.000	0.000	-0.046	0.327	0.316	0.185	0.093	0.101	0.108	-0.219
	Manufacturing	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.457	-0.405	-0.465	-0.513	-0.505	-0.525	-0.067
	Energy	0.362	0.414	0.360	0.297	0.254	0.258	-0.104	0.477	0.531	0.485	0.452	0.450	0.451	-0.027
	Hi Tech	0.000	0.016	0.018	0.045	0.005	0.000	0.000	0.061	0.149	0.121	0.114	0.114	0.112	0.051
	Telecommunications	0.286	0.151	0.084	0.045	0.000	0.000	-0.286	0.326	0.184	0.159	0.134	0.126	0.127	-0.199
	Shops/Distribution	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.047	-0.037	0.078	0.129	0.131	0.138	0.185
	Health	0.291	0.339	0.362	0.208	0.078	0.063	-0.228	0.343	0.379	0.463	0.231	0.233	0.250	-0.093
	Utilities	0.015	0.000	0.000	0.000	0.000	0.000	-0.015	0.069	0.078	0.004	-0.049	-0.051	-0.051	-0.120
	Other	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.193	-0.350	-0.490	-0.477	-0.471	-0.470	-0.276
		Two-State Model (Current State: Bear/High Volatility)													
Mean-Variance Preferences	Non Durables	0.000	0.000	0.054	0.267	0.405	0.516	0.516	-0.045	-0.217	0.632	0.748	0.844	0.945	0.990
	Durables	0.000	0.000	0.056	0.103	0.094	0.000	0.000	0.188	0.199	0.002	0.004	0.003	0.007	-0.181
	Manufacturing	0.000	0.000	0.000	0.000	0.023	0.026	0.026	1.662	2.059	0.278	0.150	0.009	-0.140	-1.802
	Energy	0.612	0.817	0.679	0.397	0.251	0.253	-0.359	0.713	0.958	0.506	0.380	0.378	0.380	-0.333
	Hi Tech	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.329	-0.048	-0.130	-0.126	-0.117	-0.122	0.207
	Telecommunications	0.239	0.000	0.000	0.000	0.000	0.005	-0.234	0.852	0.413	0.549	0.478	0.406	0.367	-0.485
	Shops/Distribution	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.144	-0.730	-0.358	-0.354	-0.306	-0.253	0.891
	Health	0.148	0.183	0.211	0.233	0.227	0.201	0.053	0.251	0.109	0.209	0.310	0.330	0.341	0.090
	Utilities	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.246	0.194	0.065	0.060	0.054	0.024	-0.222
	Other	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.393	-1.936	-0.752	-0.650	-0.600	-0.549	0.844
Mean-Var-Skew Preferences	Non Durables	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.010	-0.056	0.401	0.576	1.079	1.314	1.304
	Durables	0.246	0.088	0.113	0.224	0.184	0.145	-0.101	0.290	0.328	0.103	0.062	0.007	-0.024	-0.314
	Manufacturing	0.000	0.025	0.000	0.000	0.000	0.000	0.000	0.962	1.779	0.298	0.265	0.185	0.015	-0.947
	Energy	0.209	0.766	0.804	0.676	0.603	0.559	0.350	0.550	0.943	0.427	0.451	0.470	0.297	-0.253
	Hi Tech	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.210	-0.014	-0.100	-0.139	-0.135	-0.155	0.055
	Telecommunications	0.000	0.000	0.039	0.000	0.000	0.000	0.000	0.855	0.482	0.608	0.570	0.485	0.559	-0.296
	Shops/Distribution	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.931	-0.779	-0.477	-0.287	-0.296	-0.214	0.717
	Health	0.545	0.121	0.001	0.066	0.203	0.296	-0.249	0.213	0.010	0.057	0.242	0.172	0.272	0.059
	Utilities	0.000	0.000	0.043	0.034	0.010	0.000	0.000	0.216	0.196	0.084	0.075	0.073	0.004	-0.212
	Other	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.038	-1.621	-0.566	-0.558	-0.647	-0.541	0.496

Panel B (CRSP Industry Returns, 1926:07 - 2008:07)

Preferences	Industry	T=1	T=3	T=12	T=24	T=60	T=120	"Slope"	T=1	T=3	T=12	T=24	T=60	T=120	"Slope"
		Two-State Model (Current State: Bear/High Volatility)													
		No-short sales							Unconstrained						
Mean-Var-Kurtosis Preferences	Non Durables	0.000	0.000	0.095	0.356	0.502	0.515	0.515	0.088	0.259	0.450	0.630	0.736	0.750	0.662
	Durables	0.104	0.113	0.125	0.140	0.124	0.084	-0.020	0.321	0.275	0.084	-0.002	0.007	0.008	-0.313
	Manufacturing	0.000	0.000	0.000	0.000	0.037	0.033	0.033	-0.406	0.139	0.258	0.276	0.268	0.261	0.667
	Energy	0.276	0.398	0.432	0.495	0.310	0.251	-0.025	0.372	0.468	0.375	0.284	0.156	0.094	-0.278
	Hi Tech	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.048	0.004	-0.137	-0.128	-0.134	-0.074	-0.122
	Telecommunications	0.296	0.213	0.103	0.008	0.000	0.000	-0.296	0.320	0.405	0.541	0.495	0.458	0.375	0.055
	Shops/Distribution	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.041	-0.257	-0.358	-0.357	-0.348	-0.300	-0.259
	Health	0.167	0.216	0.245	0.001	0.026	0.117	-0.051	0.197	0.169	0.444	0.494	0.334	0.209	0.012
	Utilities	0.157	0.060	0.000	0.000	0.000	0.000	-0.157	0.191	0.174	0.084	0.055	0.062	0.062	-0.129
	Other	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.090	-0.637	-0.741	-0.746	-0.540	-0.385	-0.295
Mean-Var-Skew-Kurtosis	Non Durables	0.000	0.000	0.000	0.345	0.501	0.518	0.518	0.085	0.143	0.379	0.550	0.694	0.736	0.651
	Durables	0.107	0.091	0.000	0.000	0.000	0.000	-0.107	0.335	0.285	0.214	0.074	0.007	-0.034	-0.369
	Manufacturing	0.000	0.000	0.000	0.000	0.030	0.018	0.018	-0.445	0.110	0.276	0.278	0.260	0.257	0.702
	Energy	0.267	0.406	0.535	0.614	0.305	0.287	0.020	0.360	0.483	0.382	0.356	0.295	0.178	-0.182
	Hi Tech	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.066	0.047	-0.064	-0.125	-0.128	-0.126	-0.193
	Telecommunications	0.297	0.230	0.074	0.000	0.000	0.000	-0.297	0.321	0.368	0.544	0.538	0.543	0.526	0.205
	Shops/Distribution	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.028	-0.285	-0.358	-0.360	-0.305	-0.140	-0.112
	Health	0.163	0.228	0.391	0.041	0.165	0.177	0.014	0.200	-0.041	-0.086	0.084	0.124	0.173	-0.026
	Utilities	0.166	0.045	0.000	0.000	0.000	0.000	-0.166	0.206	0.140	0.064	0.061	0.042	-0.023	-0.229
	Other	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.099	-0.251	-0.351	-0.456	-0.533	-0.548	-0.448
		Two-State Model (Current State: Bull/Low Volatility)													
Mean-Variance Preferences	Non Durables	0.171	0.115	0.095	0.476	0.675	0.683	0.512	1.616	2.515	0.950	0.963	0.962	0.974	-0.642
	Durables	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.316	0.497	0.114	0.111	0.103	0.106	-0.210
	Manufacturing	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-2.011	-2.570	-0.609	-0.581	-0.586	-0.588	1.422
	Energy	0.218	0.147	0.049	0.154	0.261	0.260	0.042	0.960	1.302	0.466	0.447	0.449	0.449	-0.512
	Hi Tech	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.355	0.528	0.124	0.129	0.135	0.138	-0.218
	Telecommunications	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.531	-1.471	-0.405	0.053	0.102	0.112	0.643
	Shops/Distribution	0.195	0.049	0.000	0.000	0.000	0.000	-0.195	0.698	0.795	0.350	0.215	0.175	0.166	-0.531
	Health	0.416	0.689	0.856	0.370	0.064	0.058	-0.359	0.708	1.084	0.510	0.154	0.153	0.142	-0.567
	Utilities	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.667	-1.003	-0.062	-0.043	-0.049	-0.056	0.611
	Other	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.445	-0.678	-0.438	-0.448	-0.444	-0.443	0.002
Mean-Var-Skew Preferences	Non Durables	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.234	0.976	0.773	0.906	1.180	1.126	-0.108
	Durables	0.865	0.259	0.000	0.000	0.000	0.000	-0.865	0.359	0.613	0.117	0.094	0.023	-0.046	-0.406
	Manufacturing	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.087	-1.542	-0.706	-0.552	-0.624	-0.549	0.538
	Energy	0.000	0.245	0.453	0.311	0.333	0.295	0.295	1.037	1.374	0.431	0.401	0.232	0.257	-0.781
	Hi Tech	0.000	0.000	0.035	0.075	0.146	0.205	0.205	0.453	0.377	0.174	0.146	0.083	0.158	-0.295
	Telecommunications	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.087	-0.630	-0.165	0.050	0.034	0.017	0.105
	Shops/Distribution	0.135	0.049	0.005	0.000	0.000	0.000	-0.135	0.566	0.610	0.358	0.266	0.277	0.207	-0.359
	Health	0.000	0.447	0.507	0.614	0.521	0.500	0.500	0.601	0.613	0.508	0.156	0.236	0.243	-0.358
	Utilities	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.248	-0.507	-0.068	-0.051	-0.070	-0.082	0.166
	Other	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.322	-0.496	-0.532	-0.336	-0.410	-0.556	-0.234
Mean-Var-Kurtosis Preferences	Non Durables	0.000	0.000	0.000	0.240	0.604	0.677	0.677	-0.086	0.142	0.892	0.864	0.879	0.871	0.957
	Durables	0.126	0.063	0.004	0.000	0.000	0.000	-0.126	0.275	0.285	0.118	0.108	0.117	0.110	-0.165
	Manufacturing	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.469	-0.703	-0.608	-0.583	-0.585	-0.594	-0.125
	Energy	0.263	0.331	0.378	0.324	0.288	0.267	0.004	0.349	0.457	0.447	0.456	0.448	0.456	0.108
	Hi Tech	0.075	0.020	0.000	0.000	0.000	0.000	-0.075	0.237	0.248	0.140	0.125	0.127	0.144	-0.094
	Telecommunications	0.212	0.154	0.064	0.014	0.000	0.000	-0.212	0.222	0.141	0.093	0.112	0.104	0.105	-0.117
	Shops/Distribution	0.045	0.120	0.084	0.033	0.000	0.000	-0.045	0.205	0.209	0.154	0.177	0.172	0.169	-0.036
	Health	0.167	0.284	0.470	0.389	0.108	0.056	-0.110	0.280	0.344	0.250	0.238	0.249	0.249	-0.031
	Utilities	0.113	0.028	0.000	0.000	0.000	0.000	-0.113	0.169	0.060	-0.051	-0.059	-0.058	-0.053	-0.222
	Other	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.181	-0.183	-0.437	-0.438	-0.454	-0.456	-0.276
Mean-Var-Skew-Kurtosis	Non Durables	0.000	0.000	0.000	0.359	0.674	0.686	0.686	-0.099	0.133	0.350	0.604	0.864	0.870	0.969
	Durables	0.130	0.070	0.000	0.000	0.000	0.000	-0.130	0.292	0.304	0.112	0.059	-0.094	-0.145	-0.437
	Manufacturing	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.505	-0.669	-0.568	-0.350	-0.246	-0.146	0.359
	Energy	0.258	0.394	0.356	0.285	0.262	0.259	0.001	0.339	0.510	0.446	0.145	0.004	-0.024	-0.363
	Hi Tech	0.091	0.013	0.000	0.000	0.000	0.000	-0.091	0.253	0.226	0.168	0.115	0.076	0.054	-0.199
	Telecommunications	0.201	0.166	0.085	0.029	0.000	0.000	-0.201	0.226	0.162	0.096	0.034	-0.024	-0.095	-0.321
	Shops/Distribution	0.046	0.071	0.039	0.000	0.000	0.000	-0.046	0.218	0.186	0.183	0.165	0.174	0.174	-0.045
	Health	0.154	0.269	0.520	0.327	0.064	0.056	-0.099	0.287	0.335	0.250	0.243	0.253	0.249	-0.038
	Utilities	0.119	0.018	0.000	0.000	0.000	0.000	-0.119	0.176	0.042	-0.042	-0.047	-0.050	-0.056	-0.232
	Other	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.188	-0.229	-0.357	-0.360	-0.305	-0.270	-0.082

Table 6 (continued)

Portfolio Weights as a Function of the Initial State

Panel C (International Book-to-Market Sorted Portfolio Local Returns, 1975:01 - 2007:12)

		T=1	T=3	T=12	T=24	T=60	T=120	"Slope"	T=1	T=3	T=12	T=24	T=60	T=120	"Slope"
		No-short sales							Unconstrained						
		Single-State Model (Unconditional Allocation)													
	World	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-4.900	-3.654	-3.659	-3.483	-2.956	-2.669	2.231
	EU ex-UK ex-Scand Value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.372	1.254	0.649	0.570	0.528	0.485	-0.887
	EU ex-UK ex-Scand Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.910	-0.954	0.018	0.495	0.869	0.984	2.894
	United Kingdom Value	0.599	0.677	0.781	0.596	0.240	0.000	-0.599	1.725	1.405	0.749	0.675	0.304	0.251	-1.475
	United Kingdom Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.282	-0.957	-0.382	-0.362	-0.383	-0.370	-0.087
	Asia & Pacific Value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2.731	3.236	1.059	0.845	0.784	0.753	-1.978
	Asia & Pacific Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-2.678	-1.495	-0.658	-0.345	-0.084	0.149	2.827
	Scandinavia Value	0.401	0.323	0.219	0.094	0.000	0.000	-0.401	-0.044	-0.694	-0.442	-0.433	-0.374	-0.305	-0.261
	Scandinavia Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2.057	1.743	1.345	0.850	0.274	0.129	-1.928
	United States Value	0.000	0.000	0.000	0.304	0.594	0.653	0.653	0.066	-1.234	-0.244	-0.094	0.384	0.576	0.510
	United States Growth	0.000	0.000	0.000	0.006	0.166	0.347	0.347	2.862	2.351	2.565	2.282	1.654	1.017	-1.846
		Two-State Model (Current State: Ergodic/Unconditional Probabilities)													
Mean-Variance Preferences	World	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-5.059	-3.954	-3.145	-2.759	-2.460	-2.049	3.010
	EU ex-UK ex-Scand Value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.347	1.848	0.897	0.845	0.794	0.746	-0.601
	EU ex-UK ex-Scand Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.934	-1.450	-0.748	-0.240	0.004	0.137	2.070
	United Kingdom Value	0.649	0.633	0.329	0.085	0.000	0.000	-0.649	1.728	1.143	0.648	0.539	0.375	0.204	-1.524
	United Kingdom Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.290	-1.238	-0.847	-0.704	-0.633	-0.516	-0.226
	Asia & Pacific Value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2.754	2.095	1.560	1.048	0.986	0.934	-1.820
	Asia & Pacific Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-3.113	-1.795	-1.059	-0.745	-0.548	-0.375	2.738
	Scandinavia Value	0.351	0.367	0.671	0.405	0.047	0.000	-0.351	-0.074	-0.495	-0.565	-0.575	-0.598	-0.603	-0.529
	Scandinavia Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.974	1.243	0.746	0.249	-0.249	-0.262	-2.236
	United States Value	0.000	0.000	0.000	0.435	0.847	0.906	0.906	0.127	-0.204	-0.495	0.345	0.859	1.743	1.616
	United States Growth	0.000	0.000	0.000	0.075	0.106	0.094	0.094	3.540	3.807	4.008	2.997	2.469	1.041	-2.499
Mean-Var-Skew Preferences	World	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-3.243	-3.038	-3.043	-2.974	-2.756	-2.635	0.608
	EU ex-UK ex-Scand Value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.575	0.461	0.742	0.771	0.815	0.868	0.293
	EU ex-UK ex-Scand Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.123	-0.935	-0.483	-0.043	0.065	0.141	1.264
	United Kingdom Value	1.000	1.000	0.435	0.349	0.174	0.058	-0.942	1.269	1.055	0.836	0.650	0.379	0.220	-1.048
	United Kingdom Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.055	-0.608	-0.536	-0.454	-0.339	-0.218	-0.163
	Asia & Pacific Value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.899	2.293	1.304	1.145	0.951	0.886	-1.013
	Asia & Pacific Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.587	-1.077	-0.495	-0.363	-0.148	-0.106	1.481
	Scandinavia Value	0.000	0.000	0.259	0.506	0.648	0.704	0.704	-0.055	-0.288	-0.458	-0.532	-0.600	-0.634	-0.578
	Scandinavia Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.177	0.623	0.346	0.050	-0.242	-0.295	-1.472
	United States Value	0.000	0.000	0.000	0.059	0.150	0.230	0.230	0.465	0.503	0.646	0.982	1.918	2.085	1.620
	United States Growth	0.000	0.000	0.306	0.086	0.028	0.008	0.008	1.679	2.011	2.142	1.768	0.956	0.687	-0.992
Mean-Var-Kurtosis Preferences	World	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.745	-2.469	-3.059	-3.596	-3.857	-3.950	-2.205
	EU ex-UK ex-Scand Value	0.003	0.000	0.000	0.000	0.000	0.000	-0.003	0.246	0.720	0.881	0.875	0.890	0.897	0.651
	EU ex-UK ex-Scand Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.093	0.104	0.136	0.154	0.184	0.193	0.100
	United Kingdom Value	0.097	0.343	0.294	0.154	0.035	0.000	-0.097	0.171	0.676	0.150	0.067	0.049	0.028	-0.143
	United Kingdom Growth	0.095	0.000	0.000	0.000	0.000	0.000	-0.095	0.110	-0.078	-0.385	-0.490	-0.514	-0.530	-0.640
	Asia & Pacific Value	0.146	0.000	0.000	0.000	0.000	0.000	-0.146	0.244	0.705	1.015	0.999	0.989	0.998	0.754
	Asia & Pacific Growth	0.169	0.084	0.000	0.000	0.000	0.000	-0.169	0.492	0.227	0.214	0.209	0.206	0.195	-0.297
	Scandinavia Value	0.240	0.420	0.553	0.394	0.145	0.095	-0.145	0.166	0.085	-0.405	-0.591	-0.609	-0.634	-0.800
	Scandinavia Growth	0.080	0.000	0.000	0.000	0.000	0.000	-0.080	0.102	0.035	-0.182	-0.209	-0.208	-0.242	-0.345
	United States Value	0.078	0.000	0.000	0.384	0.794	0.884	0.806	0.782	1.084	1.843	2.045	2.305	2.475	1.693
	United States Growth	0.090	0.153	0.153	0.068	0.026	0.021	-0.069	0.338	-0.089	0.793	1.537	1.566	1.570	1.231
Mean-Var-Skew-Kurtosis	World	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-2.218	-3.045	-3.345	-3.875	-3.985	-4.140	-1.922
	EU ex-UK ex-Scand Value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.045	0.212	0.670	0.748	0.894	0.913	0.958
	EU ex-UK ex-Scand Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.164	-0.809	-0.059	0.095	0.116	0.161	0.325
	United Kingdom Value	0.229	0.541	0.274	0.134	0.034	0.000	-0.229	0.548	1.359	0.854	0.745	0.459	0.274	-0.274
	United Kingdom Growth	0.082	0.000	0.000	0.000	0.000	0.000	-0.082	0.197	0.082	-0.145	-0.257	-0.450	-0.491	-0.688
	Asia & Pacific Value	0.275	0.084	0.000	0.000	0.000	0.000	-0.275	0.772	1.827	1.018	1.094	1.064	1.007	0.234
	Asia & Pacific Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.147	-0.330	-0.084	0.074	0.156	0.187	0.040
	Scandinavia Value	0.151	0.259	0.515	0.294	0.114	0.037	-0.114	-0.041	-0.175	-0.305	-0.475	-0.567	-0.607	-0.566
	Scandinavia Growth	0.228	0.047	0.000	0.000	0.000	0.000	-0.228	0.463	0.174	-0.183	-0.236	-0.217	-0.255	-0.718
	United States Value	0.000	0.037	0.200	0.560	0.674	0.749	0.749	0.734	1.145	1.748	1.904	2.456	2.955	2.221
	United States Growth	0.035	0.032	0.011	0.012	0.178	0.214	0.179	0.606	0.693	0.830	0.894	0.953	0.997	0.390
		Two-State Model (Current State: World Bull/Low Volatility)													
Mean-Variance Preferences	World	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-3.405	-3.004	-2.748	-2.244	-2.009	-1.780	1.625
	EU ex-UK ex-Scand Value	0.807	0.495	0.245	0.094	0.035	0.024	-0.783	4.277	3.884	2.364	1.303	0.984	0.967	-3.310
	EU ex-UK ex-Scand Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.868	-0.847	-0.394	-0.034	0.064	0.146	2.014
	United Kingdom Value	0.000	0.063	0.000	0.000	0.000	0.000	0.000	1.381	0.840	0.374	0.073	0.021	-0.072	-1.453
	United Kingdom Growth	0.000	0.034	0.000	0.000	0.000	0.000	0.000	-0.909	-1.450	-0.563	-0.602	-0.549	-0.548	0.361
	Asia & Pacific Value	0.109	0.084	0.063	0.005	0.000	0.000	-0.109	2.708	1.330	1.093	1.080	1.063	1.009	-1.699
	Asia & Pacific Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-4.535	-2.048	-0.749	-0.394	-0.093	0.074	4.609
	Scandinavia Value	0.074	0.056	0.000	0.000	0.000	0.000	-0.074	0.762	2.096	-0.727	-0.734	-0.696	-0.718	-1.480
	Scandinavia Growth	0.009	0.034	0.000	0.000	0.000	0.000	-0.009	-0.423	0.044	-0.453	-0.440	-0.441	-0.434	-0.011
	United States Value	0.000	0.047	0.103	0.348	0.684	0.847	0.847	2.009	0.050	3.370	3.074	2.794	1.745	-0.264
	United States Growth	0.001	0.187	0.589	0.553	0.281	0.129	0.128	1.002	0.693	-0.567	0.894	0.953	0.611	-0.391
Mean-Var-Skew Preferences	World	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-2.902	-3.018	-2.703	-1.919	-2.252	-1.932	0.970
	EU ex-UK ex-Scand Value	0.204	0.603	0.365	0.184	0.145	0.084	-0.120	2.012	2.614	1.869	1.052	0.850	0.560	-1.452
	EU ex-UK ex-Scand Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.946	-0.554	-0.099	0.074	0.250	0.483	1.429
	United Kingdom Value	0.000	0.084	0.345	0.249	0.135	0.074	0.074	0.643	0.705	0.288	0.077	-0.019	-0.304	-0.947
	United Kingdom Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.371	-0.643	-0.383	-0.569	-0.249	-0.086	0.285
	Asia & Pacific Value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.772	1.287	1.094	1.086	0.952	0.958	-0.814
	Asia & Pacific Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-2.188	-1.083	-0.583	-0.202	-0.001	0.099	2.287
	Scandinavia Value	0.000	0.256	0.290	0.453	0.506	0.629	0.629	0.371	0.953	-0.658	-0.636	-0.812	-0.727	-1.098
	Scandinavia Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.165	0.103	-0.594	-0.272	-0.339	-0.409	-0.244
	United States Value	0.000	0.000	0.000	0.084	0.194	0.204	0.204	1.580	0.677	2.323	2.675	2.480	1.999	0.419
	United States Growth	0.796	0.000	0.000	0.030	0.020	0.009	-0.787	1.193	0.693	0.446	0.894	0.953	0.358	-0.834

Panel C (International Book-to-Market Sorted Portfolio Local Returns, 1975:01 - 2007:12)

		T=1	T=3	T=12	T=24	T=60	T=120	"Slope"	T=1	T=3	T=12	T=24	T=60	T=120	"Slope"
		Two-State Model (Current State: World Bull/Low Volatility)													
		No-short sales							Unconstrained						
Mean-Var-Kurtosis Preferences	World	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.972	-2.561	-2.994	-3.204	-3.385	-3.417	-1.445
	EU ex-UK ex-Scand Value	0.092	0.424	0.355	0.264	0.104	0.048	-0.044	0.412	1.075	0.977	0.962	0.923	0.864	0.452
	EU ex-UK ex-Scand Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.118	0.195	0.178	0.162	0.147	0.142	0.024
	United Kingdom Value	0.110	0.085	0.039	0.000	0.000	0.000	-0.110	0.180	0.524	0.333	0.084	0.054	0.006	-0.174
	United Kingdom Growth	0.090	0.039	0.000	0.000	0.000	0.000	-0.090	0.143	-0.053	-0.377	-0.487	-0.532	-0.560	-0.703
	Asia & Pacific Value	0.244	0.149	0.084	0.009	0.000	0.000	-0.244	0.379	0.722	0.855	0.980	1.048	1.070	0.691
	Asia & Pacific Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.346	0.229	0.112	0.078	0.044	0.009	-0.337
	Scandinavia Value	0.150	0.254	0.428	0.301	0.173	0.095	-0.055	0.080	0.134	-0.294	-0.648	-0.698	-0.721	-0.801
	Scandinavia Growth	0.107	0.049	0.004	0.000	0.000	0.000	-0.107	0.089	0.145	-0.371	-0.400	-0.416	-0.428	-0.517
	United States Value	0.178	0.000	0.000	0.305	0.594	0.659	0.481	0.907	1.901	2.405	2.894	2.914	3.074	2.167
	United States Growth	0.029	0.000	0.090	0.121	0.129	0.198	0.169	0.318	0.693	0.176	0.894	0.953	0.961	0.642
Mean-Var-Skew-Kurtosis	World	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-3.006	-2.748	-2.405	-2.124	-1.984	-1.874	1.132
	EU ex-UK ex-Scand Value	0.097	0.310	0.094	0.048	0.005	0.000	-0.097	0.404	0.826	0.924	0.970	0.981	0.988	0.584
	EU ex-UK ex-Scand Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.117	-0.158	0.154	0.181	0.240	0.265	0.148
	United Kingdom Value	0.121	0.167	0.048	0.013	0.000	0.000	-0.121	0.182	0.506	0.235	0.083	-0.070	-0.141	-0.323
	United Kingdom Growth	0.090	0.024	0.000	0.000	0.000	0.000	-0.090	0.143	0.066	-0.204	-0.374	-0.498	-0.569	-0.712
	Asia & Pacific Value	0.254	0.135	0.084	0.056	0.005	0.000	-0.254	0.381	1.168	1.064	1.009	0.964	0.904	0.523
	Asia & Pacific Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.342	-0.133	-0.043	0.067	0.090	0.117	-0.225
	Scandinavia Value	0.146	0.320	0.731	0.485	0.294	0.085	-0.061	0.078	-0.245	-0.609	-0.677	-0.710	-0.725	-0.803
	Scandinavia Growth	0.109	0.044	0.000	0.000	0.000	0.000	-0.109	0.111	0.197	0.074	-0.204	-0.316	-0.404	-0.515
	United States Value	0.184	0.000	0.038	0.384	0.624	0.684	0.500	0.905	1.304	1.758	2.084	2.156	2.460	1.555
	United States Growth	0.000	0.000	0.005	0.014	0.072	0.231	0.231	1.342	0.693	0.052	0.894	0.953	-0.021	-1.363
		Two-State Model (Current State: World Bear/High Volatility)													
Mean-Variance Preferences	World	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-2.954	-2.045	-1.450	-1.004	-0.964	-0.924	2.030
	EU ex-UK ex-Scand Value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.859	-1.349	-0.795	-0.204	0.004	0.158	2.017
	EU ex-UK ex-Scand Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-2.963	-1.649	-1.456	-1.094	-0.579	-0.205	2.758
	United Kingdom Value	0.795	0.927	0.785	0.649	0.359	0.124	-0.671	2.954	1.559	1.240	0.974	0.740	0.573	-2.381
	United Kingdom Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.232	0.571	-0.035	-0.154	-0.213	-0.404	-0.636
	Asia & Pacific Value	0.059	0.020	0.000	0.000	0.000	0.000	-0.059	2.549	2.094	1.649	1.395	1.147	1.059	-1.490
	Asia & Pacific Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.821	-0.958	-0.675	-0.539	-0.475	-0.438	1.383
	Scandinavia Value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.427	1.594	0.940	0.224	-0.153	-0.378	-0.805
	Scandinavia Growth	0.146	0.048	0.004	0.000	0.000	0.000	-0.146	2.054	1.748	1.104	0.749	0.473	0.173	-1.881
	United States Value	0.000	0.000	0.074	0.145	0.264	0.495	0.495	-1.180	-2.149	-1.049	-0.648	-0.174	0.299	1.479
	United States Growth	0.000	0.005	0.137	0.206	0.377	0.381	0.381	3.561	1.584	1.527	1.301	1.194	1.087	-2.474
Mean-Var-Skew Preferences	World	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-2.310	-1.742	-1.304	-1.291	-1.083	-1.029	1.281
	EU ex-UK ex-Scand Value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.045	-1.099	-0.707	-0.304	-0.043	0.113	1.158
	EU ex-UK ex-Scand Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.291	-0.893	-0.749	-0.707	-0.292	-0.088	1.202
	United Kingdom Value	0.668	0.856	0.804	0.645	0.386	0.174	-0.494	1.940	1.230	0.872	0.724	0.438	0.339	-1.601
	United Kingdom Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.136	0.235	-0.110	-0.206	-0.317	-0.382	-0.518
	Asia & Pacific Value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.621	1.609	1.330	1.217	1.191	1.155	-0.466
	Asia & Pacific Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.664	-0.584	-0.048	-0.003	-0.068	-0.159	0.505
	Scandinavia Value	0.169	0.074	0.114	0.148	0.358	0.564	0.395	0.319	0.782	0.411	0.043	-0.139	-0.235	-0.554
	Scandinavia Growth	0.163	0.065	0.000	0.000	0.000	0.000	-0.163	1.108	1.464	0.827	0.551	0.372	0.105	-1.003
	United States Value	0.000	0.000	0.000	0.063	0.174	0.230	0.230	-0.265	-0.779	0.270	0.797	0.799	1.050	1.315
	United States Growth	0.000	0.005	0.082	0.144	0.082	0.032	0.032	1.450	0.778	0.209	0.179	0.143	0.131	-1.319
Mean-Var-Kurtosis Preferences	World	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.733	-2.403	-2.954	-3.184	-3.404	-3.451	-1.718
	EU ex-UK ex-Scand Value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.268	-0.688	-0.548	-0.383	-0.143	0.088	0.356
	EU ex-UK ex-Scand Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.092	-0.182	-0.129	-0.114	-0.050	0.033	-0.059
	United Kingdom Value	0.160	0.653	0.806	0.685	0.453	0.215	0.055	0.398	1.285	0.639	0.583	0.403	0.327	-0.071
	United Kingdom Growth	0.116	0.053	0.024	0.000	0.000	0.000	-0.116	0.077	-0.094	-0.211	-0.192	-0.190	-0.194	-0.271
	Asia & Pacific Value	0.205	0.073	0.011	0.000	0.000	0.000	-0.205	0.363	1.039	1.218	1.234	1.217	1.200	0.837
	Asia & Pacific Growth	0.068	0.014	0.000	0.000	0.000	0.000	-0.068	0.380	-0.056	-0.003	0.032	0.059	0.084	-0.296
	Scandinavia Value	0.150	0.104	0.094	0.124	0.099	0.095	-0.055	0.192	0.083	-0.125	-0.145	-0.194	-0.353	-0.545
	Scandinavia Growth	0.186	0.102	0.034	0.005	0.000	0.000	-0.186	0.355	0.940	0.735	0.435	0.084	-0.040	-0.395
	United States Value	0.000	0.000	0.024	0.135	0.345	0.684	0.684	0.598	0.873	1.640	1.943	2.094	2.140	1.542
	United States Growth	0.114	0.001	0.007	0.051	0.103	0.006	-0.108	0.545	0.205	0.738	0.792	1.124	1.166	0.621
Mean-Var-Skew-Kurtosis	World	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.782	-1.596	-1.430	-1.329	-1.284	-1.253	0.529
	EU ex-UK ex-Scand Value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.259	-0.655	-0.512	-0.375	-0.104	0.087	0.346
	EU ex-UK ex-Scand Growth	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.074	-0.262	-0.194	-0.132	-0.094	0.034	-0.040
	United Kingdom Value	0.163	0.524	0.723	0.695	0.595	0.524	0.361	0.401	1.300	0.651	0.450	0.164	0.084	-0.317
	United Kingdom Growth	0.121	0.074	0.023	0.000	0.000	0.000	-0.121	0.077	-0.085	-0.200	-0.314	-0.384	-0.420	-0.497
	Asia & Pacific Value	0.214	0.184	0.099	0.074	0.013	0.000	-0.214	0.361	1.123	1.205	1.104	1.074	1.003	0.642
	Asia & Pacific Growth	0.079	0.045	0.008	0.000	0.000	0.000	-0.079	0.384	-0.108	0.449	0.453	0.419	0.214	-0.170
	Scandinavia Value	0.148	0.114	0.080	0.074	0.045	0.037	-0.111	0.175	0.065	-0.125	-0.135	-0.147	-0.094	-0.269
	Scandinavia Growth	0.183	0.043	0.014	0.000	0.000	0.000	-0.183	0.384	1.350	0.742	0.495	0.174	0.008	-0.376
	United States Value	0.000	0.009	0.053	0.147	0.204	0.415	0.415	0.618	1.044	1.759	1.984	2.044	2.093	1.475
	United States Growth	0.093	0.007	0.000	0.010	0.143	0.024	-0.069	0.566	-1.177	-1.346	-1.200	-0.862	-0.756	-1.322

Table 7

Out-of-Sample Realized Performance: Sharpe and Sortino Ratios, and CEQ

This table reports the best, second best and third best models for the INT, IND, BMINT asset menus, in the case of no short sales and when preferences are calibrated as a Taylor expansion of a power utility function with γ = 5, over four investment horizons: T=1, 12, 60, 120. The last column reports the performance of the equally weighted benchmark strategy. We rank models according to the Sharpe Ratio (SR), the Sortino Ratio, and the Certainty Equivalent (CEQ). The symbol * indicates that the model has the same ranking when short sales are allowed. 90% confidence bands are computed using a block bootstrap with 50,000 independent draws.

Panel A (International)	Best	Second Best	Third Best	1/N
		T=1		
SR	MV(1) *	MVSK *	MVS *	1/N
	0.528	0.473	0.333	0.389(third best)
	[-0.077,1.201]	[-0.125,1.127]	[-0.281,0.990]	[-0.225,1.039]
SO RT	MV(1) *	MVSK *	MVS *	1/N
	0.777	0.662	0.460	0.561 (third best)
	[-0.117,1.776]	[-0.174,1.710]	[-0.376,1.593]	[-0.346,1.521]
CEQ	MVSK *	MV(1) *	MV(2) *	1/N
	3.910	0.183	-1.344	3.291 (second best)
	[-8.668,15.999]	[-17.651,18.236]	[-17.202,13.763]	[-8.095,14.243]
		T=12		
SR	MV(1) *	MVSK *	MV(2)	1/N
	0.653	0.546	0.485	0.467
	[0.469,0.865]	[0.362,0.755]	[0.308,0.685]	[0.281,0.675]
SO RT	MV(1)	MVSK	MV(2)	1/N
	1.244	0.925	0.841	0.843 (third best)
	[1.011,1.524]	[0.695,1.221]	[0.539,1.209]	[0.566,1.079]
CEQ	MV(2)	MV(1)	MVSK	1/N
	4.993	4.713	4.580	2.947
	[1.251,9.163]	[-0.132,10.569]	[0.107,9.461]	[-1.254,7.627]
		T=60		
SR	MVS	MV(2)	MVSK	1/N
	0.671	0.667	0.496	0.374
	[0.557,0.824]	[0.554,0.819]	[0.411,0.608]	[0.291,0.472]
SO RT	MV(1)	MVSK	MV(2)	1/N
	1.437	1.386	1.345	1.392 (second best)
	[1.193,1.896]	[1.076,1.954]	[1.060,1.976]	[1.021,1.829]
CEQ	MVS	MV(2)	MV(1)	1/N
	12.3122	12.1958	9.6215	6.4525
	[9.799,15.350]	[9.757,15.256]	[6.141,15.086]	[4.643,8.786]
		T=120		
SR	MV(1)	MVS	MV(2)	1/N *
	1.709	1.483	1.479	2.675 (best)
SO RT	MV(1) *	MVS	MV(2) *	1/N *
	4.463	2.977	2.965	5.268 (best)
CEQ	MV(1) *	MVSK	MVS	1/N
	49.921	20.330	19.573	18.728

Table 7 (continued)

Out-of-Sample Realized Performance: Sharpe and Sortino Ratios, and CEQ

Panel B (Industry)	Best	Second Best	Third Best	1/N
		T=1		
SR	MV(2)	MVSK	MV(1) *	1/N
	1.138	0.984	0.874	0.754
	[0.744,1.553]	[0.634,1.347]	[0.490,1.283]	[0.451,1.244]
SO RT	MVSK *	MV(2)	MV(1) *	1/N
	1.751	1.703	1.276	1.095
	[1.079,2.563]	[1.032,2.590]	[0.674,2.059]	[0.609,1.958]
CEQ	MV(2)	MVSK	MV(1) *	1/N
	15.086	13.755	10.867	12.655 (third best)
	[8.813,21.207]	[7.257,20.415]	[5.209,16.413]	[4.573,15.742]
		T=12		
SR	MVSK *	MV(1) *	MV(2)	1/N
	0.943	0.919	0.901	0.750
	[0.822,1.082]	[0.803,1.048]	[0.794,1.016]	[0.724,0.962]
SO RT	MV(2)	MVSK *	MV(1)	1/N
	1.745	1.570	1.527	1.395
	[1.550,1.971]	[1.370,1.789]	[1.335,1.791]	[1.219,1.634]
CEQ	MV(2)	MVSK *	MV(1)	1/N *
	12.772	12.645	12.501	11.369
	[11.137,14.454]	[10.932,14.444]	[10.793,14.248]	[9.604,13.227]
		T=60		
SR	MV (1) *	MVS	MV(2)	1/N
	0.882	0.864	0.819	0.754
	[0.803,0.983]	[0.788,0.957]	[0.751,0.904]	[0.689,0.834]
SO RT	MVSK	MV(2)	MV(1)	1/N
	1.718	1.705	1.628	1.314
	[1.594,1.874]	[1.587,1.868]	[1.493,1.809]	[1.242,1.414]
CEQ	MV(2)	MVSK	MV(1)	1/N
	18.511	18.158	17.828	15.326
	[17.294,19.859]	[16.963,19.476]	[16.561,19.244]	[14.072,16.729]
		T=120		
SR	MV(1) *	MVS	MV(2) *	1/N *
	1.071	0.972	0.796	0.894 (third best)
	[0.989,1.179]	[0.902,1.061]	[0.737,0.871]	[0.824,0.982]
SO RT	MV(1)	MVS	MVSK	1/N
	2.184	1.897	1.878	1.655
	[2.009,2.480]	[1.694,2.174]	[1.703,2.094]	[1.482,1.893]
CEQ	MV(1) *	MV(2)	MVS	1/N
	30.922	30.247	30.040	27.602
	[29.832,32.113]	[28.499,32.190]	[28.557,31.719]	[25.793,29.680]

Table 7 (continued)

Out-of-Sample Realized Performance: Sharpe and Sortino Ratios, and CEQ

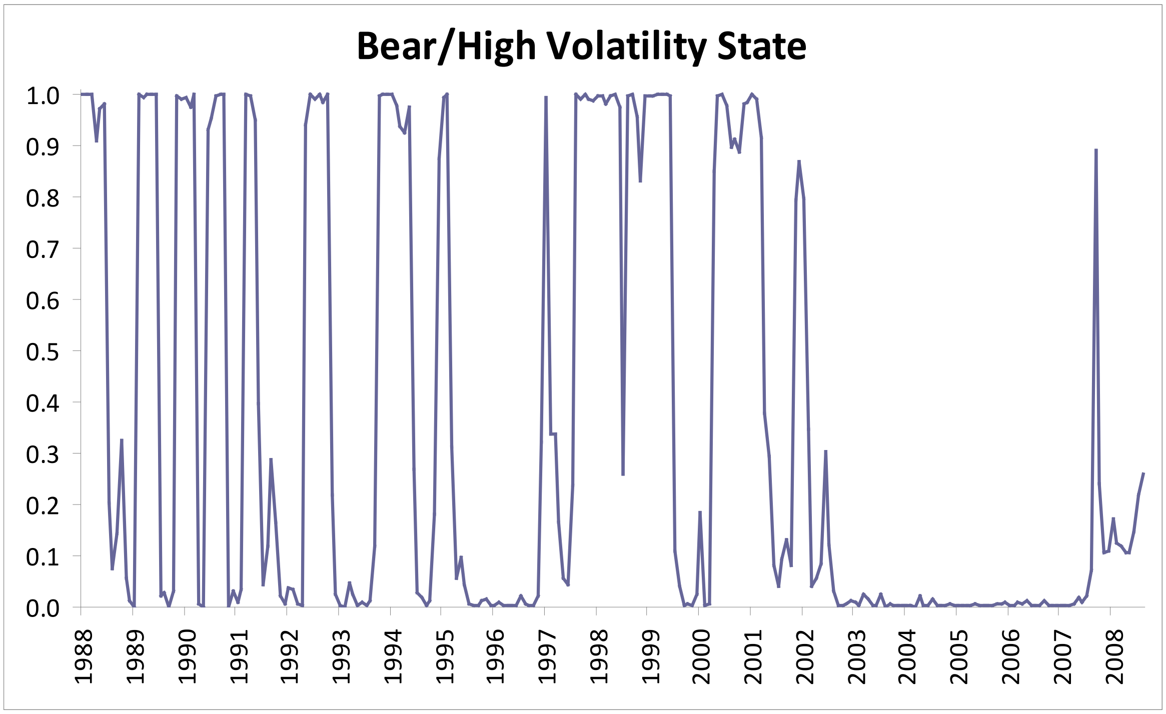
Panel C (Book-to-Market)	Best	Second Best	Third Best	1/N
		T=1		
SR	MVSK *	MV(2)	MVS*	1/N
	1.322	1.185	0.652	0.593
	[0.741,1.973]	[0.618,1.843]	[0.087,1.265]	[0.032,1.215]
SO RT	MVSK *	MV(2)	MVS	1/N
	1.857	1.586	0.897	0.790
	[1.005,2.993]	[0.811,2.689]	[0.122,1.891]	[0.043,1.704]
CEQ	MVSK *	MV(2)	MVS	1/N
	18.643	17.701	9.858	9.126
	[11.136,25.988]	[9.127,26.252]	[1.098,18.436]	[1.246,16.785]
		T=12		
SR	MVS	MV(2)	MVSK *	1/N
	0.606	0.546	0.543	0.542
	[0.415,0.836]	[0.360,0.770]	[0.367,0.750]	[0.352,0.770]
SO RT	MVS	MVSK	MV(2)	1/N
	0.791	0.751	0.666	0.686 (third best)
	[0.607,1.028]	[0.577,1.024]	[0.500,0.901]	[0.560,0.882]
CEQ	MVS	MVSK	MV(2)	1/N
	8.586	7.432	7.328	6.926
	[5.258,12.138]	[4.053,11.215]	[3.944,10.949]	[3.308,10.802]
		T=60		
SR	MVS *	MVSK *	MV(2) *	1/N
	0.571	0.186	0.169	0.280 (second best)
	[0.492,0.672]	[0.118,0.251]	[0.097,0.236]	[0.212,0.353]
SO RT	MVS *	MVSK *	MV(2) *	1/N
	1.253	0.811	0.711	0.964 (second best)
	[1.089,1.571]	[0.487,1.025]	[0.403,0.959]	[0.793,1.186]
CEQ	MVS *	MVSK *	MV(2) *	1/N
	12.868	3.603	3.378	6.469 (second best)
	[11.278,14.762]	[2.512,4.959]	[2.360,4.575]	[5.319,7.870]
		T=120		
SR	MVS	MV(2)	MVSK	1/N
	3.836	1.069	1.007	3.451 (second best)
	[3.258,4.953]	[0.902,1.396]	[0.848,1.306]	[2.692,5.323]
SO RT	MVS *	MV(2)	MVSK	1/N
	6.040	2.216	2.028	3.711 (second best)
	[4.992,9.236]	[1.810,3.284]	[1.659,2.994]	[3.040,7.034]
CEQ	MVS *	MVSK *	MV(2)	1/N
	30.206	16.165	14.964	20.768 (second best)
	[29.524,30.906]	[15.139,17.363]	[14.135,15.881]	[20.223,21.278]

Figure 1

Smoothed State Probabilities from Two-State Markov Switching Models – International Data

The graphs plot the smoothed state probabilities for the two-state switching model. Panels A, B and C respectively refer to the International, the Industry and the International Book-to-Market Portfolios.

Panel A (International MSCI USD Returns, 1988:01 - 2008:08)



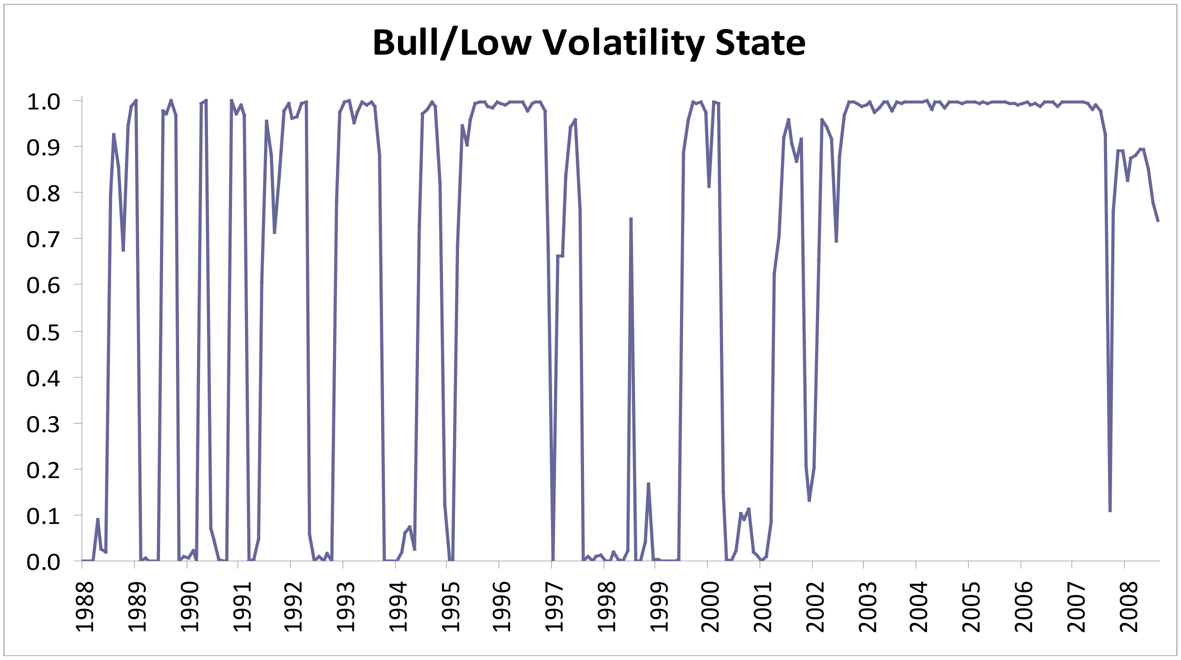
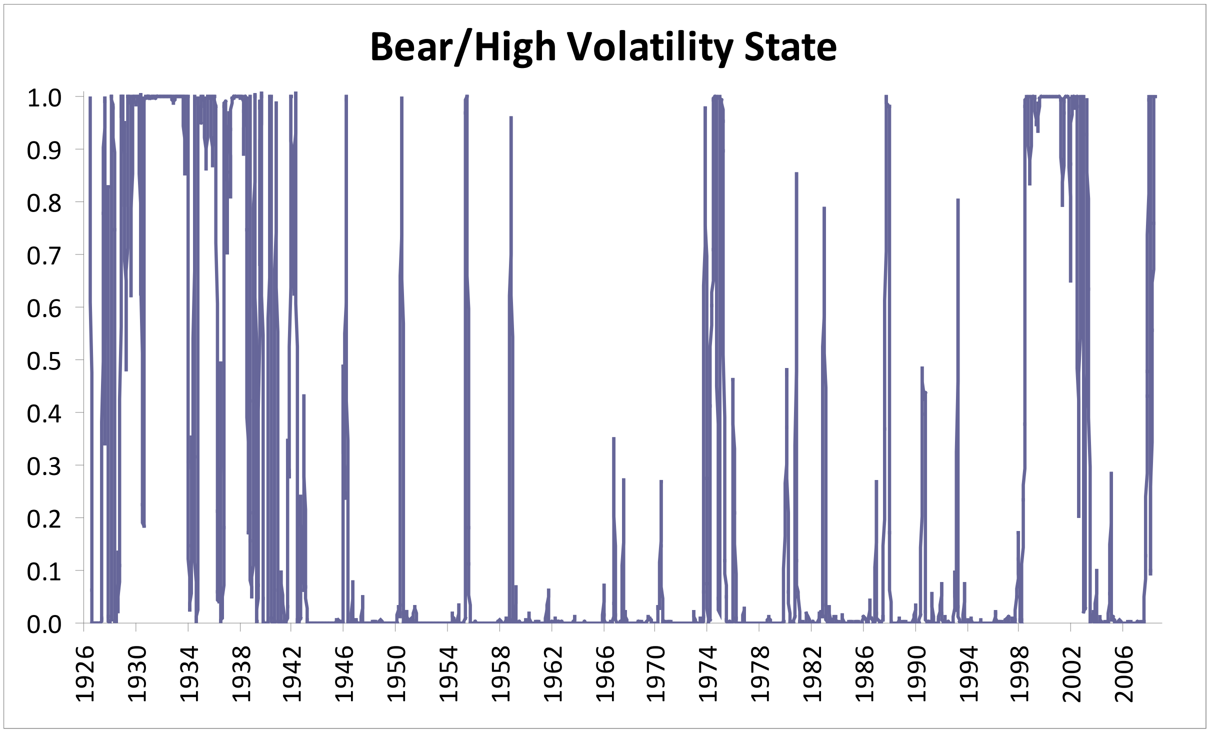


Figure 1 (cont'ed)

Smoothed State Probabilities from Two-State Markov Switching Models – Industry Data

Panel B (CRSP Industry Returns, 1926:07 - 2008:07)



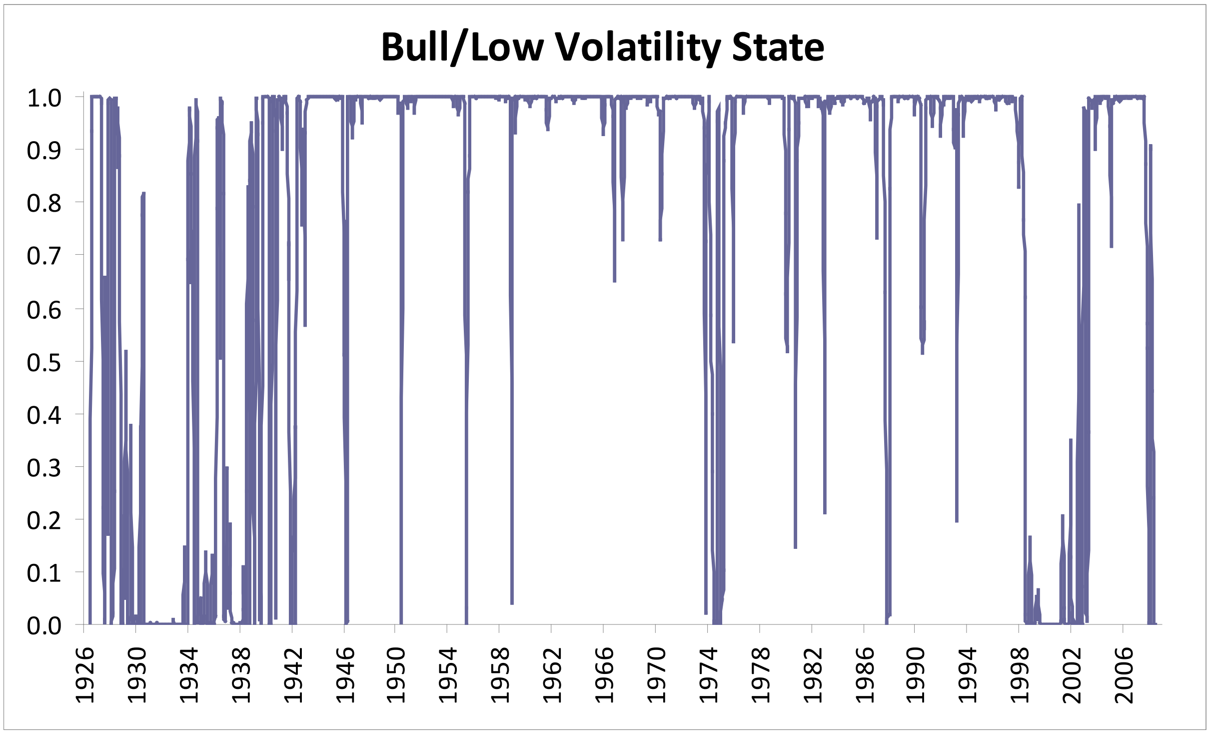


Figure 1 (cont'ed)

Smoothed State Probabilities from Two-State Markov Switching Models – Book-to-Market International Data

Panel C (International Book-to-Market Sorted Portfolio Local Returns, 1975:01 - 2007:12)